MATH 1 · LESSON 1 · IDENTITY AND INVERSE

The Do-Nothing Numbers and Their Undo Partners

A 25-minute introductory lesson for students about 12 years old. It introduces the additive and multiplicative identities, the additive and multiplicative inverses, and the parallel between them, using the proper vocabulary and plain English side by side.

Time	25 minutes (works for one student with a parent or tutor, or a small group)
Who	Math 1 (algebra readiness), about age 12
Materials	Student worksheet (PDF), the Deep Dive 5 interactive practice page for this lesson, pencil, and a whiteboard or scrap paper for the adult
Prior knowledge	Adding and subtracting negatives; multiplying fractions; has seen the word reciprocal (a quick reminder is built in)
Standards	Builds toward NC Math 1 NC.M1.A-REI.1 (justify each step when solving an equation) and NC.M1.A-REI.3 (solve linear equations). Rests on grade 6 and 7 number-system work with opposites, reciprocals, and properties of operations.

Learning goals

By the end of the lesson the student can:

- **1.** Name the **additive identity** (0) and the **multiplicative identity** (1), and explain in plain English why each one is the do-nothing number for its operation.
- **2.** Find the **additive inverse** (the opposite) and the **multiplicative inverse** (the reciprocal) of any number, and state the check: number + opposite = 0, number × reciprocal = 1.
- **3.** Spot a hidden 1 or a hidden 0 in math they already do: equivalent fractions, unit conversions, and one-step equations.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student can do it unprompted.

Term	Symbols	Plain English
Additive identity	0 $a + 0 = a$	The do-nothing number for adding. Add it and the number keeps its identity (stays itself).
Multiplicative identity	1 $a \times 1 = a$	The do-nothing number for multiplying. Multiply by it and the number stays itself.
Additive inverse (the opposite)	$-a$ $a + (-a) = 0$	The undo partner for adding. Add it to a number and you land back on 0, the do-nothing number.

Term	Symbols	Plain English
Multiplicative inverse (the reciprocal)	$\frac{1}{a}$ $a \times \frac{1}{a} = 1$	The undo partner for multiplying. Multiply a number by it and you land back on 1, the do-nothing number.
Identity property	$a + 0 = a$ $a \times 1 = a$	Adding 0 or multiplying by 1 never changes a number. Ever.
Inverse property	$a + (-a) = 0$ $a \times \frac{1}{a} = 1$	Every number has an undo partner that brings it back to the do-nothing number (except 0 has no reciprocal).

The one sentence to say out loud

An inverse is defined by the identity. The opposite of a number is whatever brings it back to zero. The reciprocal is whatever brings it back to one. If you know the do-nothing number for an operation, you already know what the undo partner has to do.

Lesson timeline (25 minutes)

- **0:00 to 0:03 Warm-up: change nothing, undo everything**

  Write these four questions and have the student answer on paper before saying anything: (a) What can I **add** to 7 so it is still 7? (b) What can I **multiply** 7 by so it is still 7? (c) What do I add to 7 to get **0**? (d) What do I multiply 7 by to get **1**?
  Expected: 0, 1, −7, $\frac{1}{7}$. Watch for −7 on question (d); that is the mix-up this lesson exists to fix.

  Say: "Two of those numbers did nothing. Two of them undid everything. Today is about those four numbers."

- **0:03 to 0:11 Mini-lesson: the parallel**

  Draw a two-column table headed **Adding** and **Multiplying**. Fill it in together, row by row, saying the term and then the plain English:
  Row 1, the do-nothing number (the identity): 0 for adding, 1 for multiplying. "Identity means it stays who it is. 7 + 0 is still 7. 7 × 1 is still 7."
  Row 2, the undo partner (the inverse): the opposite (−a) for adding, the reciprocal ($\frac{1}{a}$) for multiplying. "Inverse means turned around. It takes you back to the do-nothing number."
  Row 3, the check: $a + (-a) = 0$ and $a \times \frac{1}{a} = 1$. Point at the 0 and the 1: "See? The undo partner's whole job is to make the do-nothing number appear."

  Quick examples on the board, student answers each: opposite of 6, of −2.5, of $\frac{1}{2}$. Reciprocal of 6, of $\frac{2}{5}$, of −4, of 1. (Answers: −6, 2.5, $-\frac{1}{2}$; $\frac{1}{6}$, $\frac{5}{2}$, $-\frac{1}{4}$, 1.) Stress that the reciprocal of a negative stays negative, because $-4 \times (-\frac{1}{4}) = +1$.

  Three quick questions to end: Which number is its own opposite? (0.) Which numbers are their own reciprocal? (1 and −1.) Why does 0 have no reciprocal? (Nothing times 0 makes 1.)

- **0:11 to 0:16 The hunt for hidden ones and zeros**

  Put up three things the student already knows how to do and ask: "Where is the 1 or the 0 hiding?"

  (a) $\frac{3}{4} = \frac{9}{12}$. Hiding: we multiplied by $\frac{3}{3}$, and $\frac{3}{3}$ is 1. Nothing changed; it just looks different.

  (b) 2 feet = 24 inches. Hiding: 2 ft × $\frac{12 \text{ in}}{1 \text{ ft}}$. That fraction equals 1 because 12 inches and 1 foot are the same amount.

  (c) $x + 5 = 12$, so $x = 7$. Hiding: we added −5 to both sides. The left side became $x + 0$, and the 0 is why x is now alone.

  The line to land: "Every time you cancel, cross out, move something to the other side, or find a common denominator, you are secretly adding zero or multiplying by one. From now on we say the secret out loud."

- **0:16 to 0:22 Practice**

  Open the Deep Dive 5 interactive page for this lesson (or use Worksheet Parts A to D). The student tries each problem, then uses **Hint**, **Show next step**, and **Reveal answer** only when they want them.

  While they work, ask them to narrate: "Which do-nothing number are you aiming for? Which undo partner gets you there?" Insist on the pair: the term and the plain English.

- **0:22 to 0:25 Exit ticket**

  Worksheet Part E, or on an index card: (1) What is the reciprocal of $\frac{3}{8}$, and what do you get when you multiply them? (2) Rewrite $\frac{5}{6}$ as $\frac{?}{18}$ and circle the 1 you multiplied by. (3) One sentence: how are opposites and reciprocals alike?

  Success looks like: $\frac{8}{3}$ and 1; $\frac{15}{18}$ with $\frac{3}{3}$ circled; any sentence that says both take you back to the do-nothing number.

Misconceptions to listen for

- **Opposite and reciprocal get swapped.** "The opposite of 5 is −5; the reciprocal of 5 is $\frac{1}{5}$. Different operation, different do-nothing number, different undo partner." Send them back to the table.
- **Reciprocal of a negative comes out positive.** The reciprocal of −4 is $-\frac{1}{4}$. Check it: $-4 \times (-\frac{1}{4}) = +1$. The sign has to stay so the product can be positive 1.
- **"Multiplying by 1 changed the fraction."** $\frac{9}{12}$ looks different from $\frac{3}{4}$ but it is the same number. Equivalent fractions are the identity property wearing a disguise.
- **"Times zero does nothing, so 0 is the identity for multiplying too."** Multiplying by 0 wipes everything out; it is the most powerful change there is. The do-nothing number for multiplying is 1.
- **Reciprocal of a mixed number.** Convert first: $2\frac{1}{2} = \frac{5}{2}$, so the reciprocal is $\frac{2}{5}$, not $\frac{2}{1}$.

Support and stretch

- **Support:** Use a number line for opposites (same distance from 0, other side). Use fraction strips for reciprocals (how many $\frac{2}{5}$ strips make one whole? $\frac{5}{2}$ of them). Give the two-column table with the identities already filled in.
- **Stretch:** Is there an identity for subtraction? (0 only works on the right: 7 − 0 = 7, but 0 − 7 is not 7, so no.) Find every number that is its own reciprocal. Explain, in writing, why 0 cannot have a reciprocal.
- **Preview:** Later in Math 1, adding zero shows up when completing the square, and multiplying by one shows up in unit rates and when rewriting expressions with common denominators. Same two tricks, bigger problems.

Where this goes next

Lesson 2: solving one-step equations by naming, at every step, the inverse you used and the identity that appeared. Lesson 3: two-step equations, same narration. The Deep Dive 5 Unit Rates set is a natural companion: it solves every rate problem by multiplying by a reciprocal to turn the time into 1.