Absolute Value Inequalities: Distance From Zero

Absolute value is distance from zero. Less than a distance: between, the word is and. Greater than: outside, the word is or. Two problems, two answers, one word.

1 THE VOCABULARY
Words we use

Absolute value distance from zero, never negative: |–7| = 7 and |7| = 7. Two numbers sit at every distance, one on each side.

|inside| < 5 the inside is between –5 and 5. Two problems joined by **and**: –5 < inside **and** inside < 5. Written as a sandwich: –5 < inside < 5.

|inside| > 5 the inside is outside the two 5s. Two problems joined by **or**: inside < –5 **or** inside > 5. Never a sandwich.

And both at once: one shaded stretch between two boundaries.

Or one or the other: two pieces pointing away from each other, with a gap.

Get the bars alone peel off anything added to or multiplied on |inside| first, with the Lesson 6 moves. Flip if you multiply by a negative.

- Watch out for these four.
- Splitting before the bars are alone: 3|x + 2| + (–5) < 10 must become |x + 2| < 5 first.
- Writing the left or-problem backwards: |x| > 5 gives x < –5, not x > –5.
- Joining an or-answer as a sandwich: x < –4 or x > 2 is right; –4 < x < 2 is the gap, the exact opposite.
- A negative on the right: |inside| < –1 has no solution, |inside| > –1 is every number. Do not set up problems.

2 THE METHOD
The steps

1. **REWRITE** inside the bars and outside them: subtraction is adding the additive inverse (the opposite), |x - 3| becomes |x + (–3)|.
2. **GET THE BARS ALONE** treat |inside| as one block and peel off the rest: addition property of inequality, then multiplication property of inequality. Negative multiplier: flip now.
3. **READ THE SIGN, PICK THE WORD** less than: between, **and**. Greater than: outside, **or**.
4. **TWO PROBLEMS, SAME INSIDE, SOLVE BOTH** exactly as in Lesson 6, every line named. A negative coefficient (the number on the variable) flips both.
5. **WRITE, GRAPH, CHECK** the answer with its word. Check in the original: one number from each piece and each gap.

WHY TWO PROBLEMS
The bars hide only the sign of the inside. Close to zero can mean a little negative or a little positive (and). Far from zero can mean very negative or very positive (or). Either way the inside is the same expression, so once the bars are off you are back in Lesson 6.

3 EVERY LINE NAMED
Worked example

$\|2x + 1\| \geq 7$	the original: the distance of 2x + 1 from zero is 7 or more. Bars already alone, nothing to rewrite
$2x + 1 \leq -7$ or $2x + 1 \geq 7$	greater than a distance: outside. Two problems with the same inside, joined by **or**. The left one faces left toward –7
$2x \leq -8$ or $2x \geq 6$	add –1, the additive inverse (the opposite) of 1, to both sides of both: the addition property of inequality (add the same thing to both sides, and the sign stays the same)
$12 \times 2x \leq 12 \times (-8)$ or $12 \times 2x \geq 12 \times 6$	the multiplication property of inequality, by the multiplicative inverse (the reciprocal) of the coefficient (the number on the variable) 2. Positive, so no flip in either
$x \leq -4$ or $x \geq 3$	1x = x by the multiplicative identity (the do-nothing number 1). Two pieces pointing away from each other, closed circles at –4 and 3, gap between

CHECK THREE NUMBERS
x = –5: |–9| = 9 ≥ 7, true. x = 3, a boundary: |7| = 7 ≥ 7, true, included. x = 0, in the gap: |1| = 1 ≥ 7, false, so the gap is correctly left out.

SAY IT OUT LOUD
Distance from zero. Less than: between, and. Greater than: outside, or. Free the bars, write two problems, solve both, say the word.

Lesson 7 in one line: two problems, two answers, and the word between them is and for less than, or for greater than.