Arithmetic Sequences, Lines Made of Dots

Students find the common difference, write recursive and explicit rules for an arithmetic sequence, explain the n minus 1, connect the sequence to a line (slope d, y-intercept the zeroth term), and find which term has a given value.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 9, Lesson 1.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson, pencil, graph paper
PRIOR KNOWLEDGE	Sequences as functions (Lesson 1). Slope and slope-intercept form (Unit 3). The distributive property (Unit 1, Lesson 2). Solving two-step equations (Unit 1).
STANDARDS	NC.M1.F-IF.3, NC.M1.F-BF.1a, NC.M1.F-BF.2.
KEEP OUT	Geometric sequences (Lesson 3). Sums of sequences.

Learning goals

1. Find the common difference, including a negative one.
2. Write the recursive rule and the explicit rule, and explain why the explicit rule uses n minus 1.
3. Connect the sequence to a line: d is the slope, and the zeroth term is the y-intercept.
4. Find a far term, and find which term has a given value by solving an equation.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Arithmetic sequence	adds the same amount	Dots on a line.
Common difference	d	The slope.
Recursive rule	the stairs	Next from before.
Explicit rule	the elevator	Any term from n.
Zeroth term	$a_1 - d$	The y-intercept.

> **THE ONE SENTENCE TO SAY OUT LOUD**
> Adding gives arithmetic. The common difference is the slope. Stairs for recursive, elevator for explicit, and n minus 1 steps.

Lesson timeline (45 minutes)

0:00 to 0:05	**Warm-up: next three terms.** 60, 68, 76; then 40, 37, 34. Ask what is added each time, and name d.
0:05 to 0:18	**Mini-lesson: two rules.** Stairs and elevator. Write the recursive rule, then the explicit rule. Count steps to row 20 on fingers to get 19. Compute 212. Run the stair-step graph.
0:18 to 0:28	**It is a line.** d is the slope. Simplify 60 + 8(n − 1) to 8n + 52. Step back from 60 to find 52, the zeroth term. State the trap. Show the notation table.
0:28 to 0:33	**Backward.** Concert hall: which row has 94 seats? Equation, then Unit 1 moves.
0:33 to 0:41	**Practice.** The practice problems or Worksheet Parts A to D. Same buttons: Hint, Show next step, Reveal answer, Hide help.
0:41 to 0:45	**Exit ticket.** Worksheet Part E. Success looks like d = 7, both rules, and 215.

Misconceptions to listen for

- **Multiply d by n.** n − 1 steps: the first term is already there.
- **The first term is the y-intercept.** The y-intercept is at n = 0, one step back.
- **d is always positive.** A falling list has a negative d.
- **A recursive rule alone is enough.** It needs the starting term.
- **Connect the dots.** The line is a guide; the sequence is the dots.

Support and stretch

Support

Have the student draw the hops between terms and count them. Write the zeroth term in a box to the left of the list. Keep Unit 3's y = mx + b card on the desk.

Stretch

Give the 5th term (23) and the 9th term (39) and ask for d and the first term. Ask which term of 60, 68, 76 first passes 500.

Where this goes next

Lesson 3: geometric sequences. Multiply instead of add: the common ratio is the growth factor, and the dots sit on an exponential curve.

> THE HABIT TO BUILD
> Subtract to find d, write the start and the rule, count n minus 1 steps, and step back once for the y-intercept.