Average Rate of Change: How Fast It Is Changing

Students put a number on steeper means faster: the average rate of change, the change in the output over the change in the input across an interval, found from a table, a graph, and a rule, and read back into the story. This is the bridge to slope, which is named in Unit 3.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 2, Lesson 5 (graphing stories).
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson, pencil, calculator optional
PRIOR KNOWLEDGE	Evaluating f(x) (Lesson 1). Reading points off a graph (Lessons 3 and 4). Steeper means faster (Lesson 5). Fractions as top over bottom, and subtracting with negatives (Unit 1).
STANDARDS	NC Math 1 NC.M1.F-IF.6 (calculate and interpret the average rate of change of a function over a specified interval, from a table, a graph, or a rule). Leads into NC.M1.F-LE and slope in Unit 3.
KEEP OUT	Do not name it slope, do not write y = mx + b. Say: next unit gives this its real name.

Learning goals

1. Say what the average rate of change is in plain English: how much the output changes for each step of the input, over an interval.
2. Find it from a table, from two points read off a graph, and from a rule (evaluate first), using the same three moves: subtract the outputs, subtract the inputs in the same order, divide.
3. Interpret the sign and size: positive climbs, negative falls, zero is flat, bigger is steeper.
4. Read the number back into the story with the interval and the units.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Average rate of change	change in output over change in input	How much it went up or down for each step to the right, across a stretch.
Interval	from x = 1 to x = 5	The stretch of inputs. Always name it.
Top, bottom	numerator, denominator	Change in the output over change in the input. Second minus first, both times.
Positive, negative, zero	sign of the rate	Climbing, falling, flat.
Per	miles per hour	For each one.

> THE ONE SENTENCE TO SAY OUT LOUD
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> Two points. Outputs on top, inputs on the bottom, second minus first both times. Divide. Say the interval and the units, and say what the sign means in the story.

Lesson timeline (45 minutes)

0:00 to 0:05	Warm-up: steeper means faster. Two climbs on a distance graph, 6 blocks in 3 minutes and 6 blocks in 6 minutes. Ask which is faster and how much faster. Let the student reach for blocks per minute on their own: 2 and 1.
0:05 to 0:15	Mini-lesson: the fraction. Write it as a stacked fraction: change in the output over change in the input. Do the plant table together: two rows, top, bottom, divide, then the sentence: about 2 inches per week between week 1 and week 5. Do the same two points on the graph and show the dashed steps: over 4, up 8. Then a rule: $f(x) = x \times x + 1$ from 1 to 3, evaluating first.
0:15 to 0:25	Sign and interval. The candle gives a negative rate: say what that means. The parked car gives 0. Then the plant on the interval 0 to 2 gives 1.5: different interval, different rate, which is why we always name it. Work the two interactive graphs together (road trip, cooling cup).
0:25 to 0:40	Practice. Interactive practice problems or Worksheet Parts A to D. Same buttons: Hint, Show next step, Reveal answer, Hide help. Require units and the interval on every answer, and a story sentence for every sign.
0:40 to 0:45	Exit ticket. Worksheet Part F or an index card: the three items. Success looks like: 45 miles per hour; 2 per step; –30 gallons per hour, losing water.

Misconceptions to listen for

- **Subtracting in different orders.** Top second minus first, bottom first minus second, and the sign flips. Same order, both times.
- **Input over output.** The fraction is output change over input change: how much y for each x.
- **Dropping the sign.** A negative rate is information: falling. Keep it and say it.
- **Forgetting the interval.** On a curve, a different stretch gives a different rate. The plant is 1.5 per week early and 2 per week later.
- **Skipping evaluation with a rule.** $f(x) = x \times x$ from 1 to 3 is not 3 minus 1 over anything. Find f(1) and f(3) first.
- **Calling it slope.** Some students know the word. Say: that is its name next unit; for now, average rate of change, because it is an average over a stretch.

Support and stretch

Support

Keep the stacked fraction on the board with the words in it. Give a three-line template: points, top and bottom, divide and sentence. Use tables with friendly numbers before graphs, and straight-line rules before curved ones.

Stretch

Ask for two intervals of a curve with the same average rate. Ask what happens to the rate of a cooling cup as the interval moves later. Ask why a straight line has the same rate on every interval, and connect it back to Lesson 2.

Where this goes next

Unit 3, Linear Functions. The fraction from this lesson gets its real name, slope. Students learn $y = mx + b$, where m is this rate and b is the y-intercept from Lesson 4, and they write and graph the equation of a line. Unit 3 should feel like naming things they have already seen.

> THE HABIT TO BUILD
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> Every rate: two points, top over bottom, second minus first both times, divide, then say it with units and the interval.