Average Rate of Change: How Fast It Is Changing

Pick two points. The average rate of change is the change in the output over the change in the input: how much it went up or down for each step to the right, across that interval. Positive climbs, negative falls.

1 THE VOCABULARY
Words we use

Average rate of change how much the output changes for each step of the input, over a stretch: change in the output divided by change in the input.

Interval the stretch of inputs, from one value to another. Always say which one.

Top (numerator) the change in the output: second output minus first.

Bottom (denominator) the change in the input: second input minus first, same order.

Positive, negative, zero climbing, falling, flat.

Per for each one: miles per hour is miles for each hour.

- Watch out for these.
- Subtracting in different orders on the top and the bottom. Second minus first, both times.
- Forgetting the interval. A curve has a different rate on a different stretch.
- Dropping the sign. A negative top means the output went down: say so in story words.
- Skipping the evaluate step for a rule. Find the two outputs first, then subtract.
- Calling it slope. Not yet: Unit 3 names it and pairs it with the equation of a line.

2 THE METHOD
The three moves

1. PICK TWO POINTS ON THE INTERVAL from a table: two rows. From a graph: read two points. From a rule: evaluate at the two inputs first.
2. TOP: SUBTRACT THE OUTPUTS second minus first. This is the change in the output.
3. BOTTOM: SUBTRACT THE INPUTS second minus first, same order. This is the change in the input.
4. DIVIDE, THEN READ IT BACK the number is the change per one step. Say the units and the interval: about 2 inches per week between week 1 and week 5.

WHY IT IS CALLED AVERAGE
Between two points a curve may speed up and slow down. The fraction smooths all of that into one number for the whole stretch, as if the change had been spread out evenly. A straight line has the same rate on every interval, which is what made it straight in Lesson 2.

3 THREE WAYS
The same three moves, from a table, a graph, and a rule

table: (1, 4) and (5, 12)	the plant at week 1 and week 5. Top 12 – 4 = 8, bottom 5 – 1 = 4, rate 8/4 = 2 inches per week
graph: read (2, 5) and (6, 13)	two points off the picture. Top 13 – 5 = 8, bottom 6 – 2 = 4, rate 2 inches per week
graph, another interval: (0, 2) and (2, 5)	top 3, bottom 2, rate 1.5 inches per week. A different interval, a different rate
rule: f(x) = x × x + 1 from 1 to 3	evaluate first: f(1) = 2, f(3) = 10. Top 8, bottom 2, rate 4 per step of x
negative: candle (0, 8) to (4, 6)	top 6 – 8 = –2, bottom 4, rate –1/2 inch per hour: burning down half an inch each hour

CHECK THE SIGN AGAINST THE PICTURE
If the second point is higher than the first, the rate must be positive and the graph climbs there. If it is lower, negative and falling. If your sign disagrees with the picture, the subtraction order slipped.

SAY IT OUT LOUD
Two points. Outputs on top, inputs on the bottom, second minus first both times. Divide. Say the units and the interval.