Average Rate of Change: How Fast It Is Changing

Average rate of change: the change in the output over the change in the input, across an interval. Pick two points, subtract, subtract, divide, then read the number back into the story with units.

Name _____ Date _____

Words we use

say the math word, then the plain English

WORD	PLAIN ENGLISH	WHAT YOU DO
Average rate of change	How much the output changes for each step of the input, over a stretch. The change in the output divided by the change in the input.	Pick two points, subtract the outputs for the top, subtract the inputs for the bottom, divide.
Interval	The stretch of inputs from one value to another, like from x = 1 to x = 5.	Always say which interval you used. A curve can have a different rate on a different interval.
Change in the output	The top of the fraction (the numerator): second output minus first output.	Keep the order. A negative top means the output went down.
Change in the input	The bottom of the fraction (the denominator): second input minus first input, same order.	Usually positive when you read left to right.
Positive rate, negative rate	Positive: climbing. Negative: falling. Zero: flat.	Say the sign in story words: growing 2 inches per week, losing 30 gallons per minute.
Per	For each one. Miles per hour means miles for each hour.	Every rate is a per: output units per one input unit.

Part A From a table

For each interval, write the two points, the top (change in the output), the bottom (change in the input), and the rate with units. Row 1 is done for you.

Distance of a train: hours 0, 1, 3, 4 and miles 0, 60, 200, 200.

INTERVAL	TWO POINTS	TOP	BOTTOM	RATE
hour 0 to 1	(0, 0), (1, 60)	60 – 0 = 60	1 – 0 = 1	60 miles per hour
hour 1 to 3				
hour 3 to 4				
hour 0 to 4				

Which interval was the fastest? What was the train doing from hour 3 to hour 4?

Part B From a graph

Points read off a graph of a candle's height (hours, inches): (0, 10), (2, 8.5), (6, 5.5).

1. Rate from hour 0 to hour 2, with units and sign:

Part C From a rule

Evaluate at both inputs first, write the two points, then the three moves.

2. Rate from hour 2 to hour 6:

1. $f(x) = 2x + 3$, from $x = 1$ to $x = 4$.

3. What does the sign mean in the story?

2. $f(x) = 2x + 3$, from $x = 0$ to $x = 10$. Compare with question 1.

3. $g(x) = x \times x$, from $x = 2$ to $x = 4$, and then from $x = 4$ to $x = 6$. Compare.

Part D Read it back into the story

Write the sentence a scientist or a coach would write, with the interval and the units.

1. A puppy weighed 6 pounds at 8 weeks and 14 pounds at 16 weeks.

2. A pool held 900 gallons at 2 p.m. and 600 gallons at 5 p.m.

3. A runner was 3 miles out at minute 30 and 3 miles out at minute 40.

Part E In your own words

1. What goes on the top of the fraction, and what goes on the bottom? Why does the order of subtraction matter?

2. Why do we always say which interval we used? Give an example where two intervals have different rates.

3. What does a negative rate tell you about the graph? What does a rate of 0 tell you?

Part F Exit ticket

1. Hours 1, 3, 5 and miles 40, 130, 220: rate from hour 1 to hour 5, with units.

2. $f(x) = 2x + 5$ from $x = 1$ to $x = 4$.

3. A tank: 500 gallons at noon, 380 at 4 p.m. Rate and the meaning of the sign.

Answer key

Part A. Hour 1 to 3: (1, 60), (3, 200); top 140; bottom 2; 70 mph. Hour 3 to 4: top 0; bottom 1; 0 mph, stopped. Hour 0 to 4: top 200; bottom 4; 50 mph. Fastest: hour 1 to 3.

Part B. 1. Top 8.5 – 10 = –1.5, bottom 2, rate –0.75 inch per hour. 2. Top 5.5 – 8.5 = –3, bottom 4, rate –0.75 inch per hour. 3. Negative: the candle is getting shorter, three quarters of an inch each hour.

Part C. 1. f(1) = 5, f(4) = 11; top 6, bottom 3; rate 2. 2. f(0) = 3, f(10) = 23; top 20, bottom 10; rate 2. Same rate on both intervals: a straight line. 3. g(2) = 4, g(4) = 16, rate 6; g(4) = 16, g(6) = 36, rate 10. Different: the curve gets steeper.

Part D. 1. The puppy gained about 1 pound per week between week 8 and week 16. 2. The pool lost 100 gallons per hour between 2 and 5 p.m. 3. The runner's distance changed 0 miles per minute between minute 30 and 40: stopped.

Part E. 1. Top: change in the output; bottom: change in the input; second minus first both times, or the sign comes out wrong. 2. A curve has different rates on different stretches, like g(x) = x × x above. 3. Negative: falling. Zero: flat.

Part F. 1. 180 over 4 = 45 miles per hour. 2. f(1) = 7, f(4) = 13, rate 2. 3. –120 over 4 = –30 gallons per hour: losing water.