Compound Interest, Interest on Interest

Students build compound interest year by year, see the interest earned (the average rate of change) grow, learn why adding r is multiplying by 1 + r, write A = P(1 + r)^t, connect it to y = a · b^x, and read P and r out of an expression.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 8, Lesson 1.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson (side-by-side table and explorer need a screen), pencil, calculator with a power key
PRIOR KNOWLEDGE	Simple interest (Lesson 1). y = a · b^x, growth factor and rate (Unit 7). The distributive property (Unit 1, Lesson 2). Average rate of change (Unit 3).
STANDARDS	NC.M1.F-IF.8b, NC.M1.F-LE.5, NC.M1.A-SSE.1b, NC.M1.F-IF.6.
KEEP OUT	Compounding more than once a year (Lesson 3). Solving for the exponent.

Learning goals

1. Build a compound interest table year by year and explain why the interest earned grows.
2. Explain, with the distributive property, why adding r is multiplying by 1 + r.
3. Write and evaluate A = P(1 + r)^t to the cent, power first, and match it to y = a · b^x.
4. Read the principal and the rate from an expression such as 10,000(1.055)^n, and explain why the balance is never 0.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Compound interest	interest on interest	The balance multiplies.
Compounded annually	once a year	One factor per year.
Growth factor	$1 + r$	What you multiply by.
Growth rate	r	Distance of the factor from 1.
Initial value	P	The balance at $t = 0$.

> THE ONE SENTENCE TO SAY OUT LOUD
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> Simple interest adds, compound interest multiplies. The growth factor is 1 plus the rate: keep all of it, and add the interest.

Lesson timeline (45 minutes)

0:00 to 0:05	**Warm-up: Lesson 1's table.** $1,000 at 5% simple: 1,000, 1,050, 1,100. Ask: what if the $50 stayed in the account and earned interest too?
0:05 to 0:17	**Mini-lesson: interest on interest.** Build the compound table with the interest column: $50.00, $52.50, $55.13. Name the growing rate of change. Run the side-by-side builder and watch compound pull ahead.
0:17 to 0:28	**The shortcut and the formula.** $B + 0.05B = B(1.05)$. Every year multiplies by 1.05. $A = P(1 + r)^t$, next to $y = a \cdot b^x$. Check year 3 with the calculator keys. Then read $10{,}000(1.055)^n$ backward and ask whether the balance can be 0. Explorer.
0:28 to 0:40	**Practice.** The practice problems or Worksheet Parts B to D. Same buttons: Hint, Show next step, Reveal answer, Hide help. Power first every time.
0:40 to 0:45	**Exit ticket.** Worksheet Part F. Success looks like $3,000, 1.07, 7%; and $562.43.

Misconceptions to listen for

- **The factor is 0.05.** That would keep only 5%. The factor is 1.05.
- **1.055 means 55%.** 1 + 0.055: 5.5%.
- **Multiply P by 1.05, then raise to the power.** Power first.
- **Compound is a steeper line.** The interest grows each year: not a line.
- **The balance could reach 0.** Positive times positive stays positive.

Support and stretch

Support

Keep the year-by-year table on the desk and check each formula answer against it. Have the student say keep all of it, plus the interest, when writing 1 + r.

Stretch

Ask how many years the compound account needs to be $500 ahead of the simple one (use the side-by-side builder). Ask what rate gives a factor of 2, and what that means.

Where this goes next

Lesson 3: compounding more often. The yearly rate is split into n pieces, r over n, and paid n times a year.

> THE HABIT TO BUILD
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> Factor first (1 plus the rate), power next, then times P. Read the front number as P and the factor as 1 plus the rate.