Distance, the Pythagorean Theorem on the Grid

Students count horizontal and vertical distances, draw the right triangle under a slanted segment, use the Pythagorean theorem, notice the repeated steps and name them as the distance formula, and answer why order does not matter and why there is a square root.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 10. Opens Unit 11, the course's geometry unit.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson, pencil, graph paper, calculator for square roots
PRIOR KNOWLEDGE	The Pythagorean theorem (8th grade, 8.G.8). Plotting points and slope (Units 3 and 4). Inverses (Unit 1, Lesson 1). Squares and square roots of whole numbers.
STANDARDS	NC.M1.G-GPE.4; prerequisite 8.G.8.
KEEP OUT	Simplifying radicals (Unit 13). Midpoint (Lesson 2). Three-dimensional distance.

Learning goals

1. Find the distance between two points on the same horizontal or vertical line by counting.
2. Draw the right triangle under a slanted segment, label the legs, and use the Pythagorean theorem.
3. State the distance formula as the pattern of those steps, and use it in all four quadrants, giving exact and decimal answers.
4. Explain why order does not matter and why the formula ends with a square root.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Distance	how long	Always positive.
Legs, hypotenuse	across and up; the slant	The right triangle under the segment.
Pythagorean theorem	$a^2 + b^2 = c^2$	Legs to hypotenuse.
Square root	undoes squaring	Exact and to the hundredth.

> THE ONE SENTENCE TO SAY OUT LOUD
>
> How long? Draw the right triangle. Across, up, square, add, square root. The square root undoes the square.

Lesson timeline (45 minutes)

0:00 to 0:05	**Warm-up: count it.** (2, 3) to (9, 3), then (4, –1) to (4, 5). Then (1, 2) to (4, 6): can you count that? No.
0:05 to 0:17	**Mini-lesson: the triangle.** Draw the legs, name the hypotenuse, apply $a^2 + b^2 = c^2$. Use the interactive tool for two more, including one with negatives. Ask: what did you do every time? Write the pattern as the formula.
0:17 to 0:25	**The three questions.** Order (squaring removes the sign). Square root (undo partner). Slope versus distance (divide versus square and add).
0:25 to 0:40	**Practice.** The practice problems or Worksheet Parts A to D. Same buttons: Hint, Show next step, Reveal answer, Hide help. Every answer exact and to the hundredth.
0:40 to 0:45	**Exit ticket.** Worksheet Part F. Success looks like 10, root 61 ≈ 7.81, and the squaring reason.

Misconceptions to listen for

- **Stopping at c^2.** Take the square root.
- **Adding the legs.** Square them first: 3 + 4 is not 5, but 9 + 16 = 25 is.
- **A negative distance.** Squaring removes the sign.
- **Simplifying the root.** Leave root 61 as it is for now.
- **Mixing up slope and distance.** Same differences; divide for slope, square and add for distance.

Support and stretch

Support

Have the student draw and label every triangle before touching the formula. Keep a list of perfect squares to 144 on the desk.

Stretch

Ask which points are exactly 5 units from the origin with whole-number coordinates (there are 12). Ask whether a distance of root 2 can be drawn on the grid.

Where this goes next

Lesson 2: the midpoint. Where is the middle? Halfway across and halfway up, the average of the coordinates.

> THE HABIT TO BUILD
>
> Draw the triangle, then across, up, square, add, square root.