Domain and Range: The Inputs and the Outputs

Students name the two piles of numbers on either side of a function: the domain (the inputs) and the range (the outputs). They find a reasonable domain and range in a story and read both off a graph, writing answers in words or as inequalities.

TIME	40 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 2, Lesson 2 (from points to a line; continuous and discrete).
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson, pencil, scrap paper
PRIOR KNOWLEDGE	A function gives one output for each input (Lesson 1). Continuous versus discrete (Lesson 2). Solving a one-step equation such as $8 - t = 0$ (Unit 1).
STANDARDS	NC Math 1 NC.M1.F-IF.5 (interpret a domain and range in context) and NC.M1.F-IF.4 (interpret key features of a graph). Domain and range are written in words or as inequalities; interval notation is not used in Math 1.
KEEP OUT	No interval notation. No slope, no equations of lines (Unit 3).

Learning goals

1. Say what the domain (the inputs) and the range (the outputs) are, and find them from a table, listing each output value once.
2. Give a reasonable domain and range for a story by asking: smallest input, largest input, whole things or any number? Then feed in the ends.
3. Read the domain and range off a graph: left to right along the bottom, bottom to top along the side, as an inequality for a connected graph or a list for separate dots.
4. Write every answer in words or as an inequality, with units.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Domain	the x values, $0 \le x \le 8$	The inputs: everything the function is allowed to take.
Range	the y values, $1 \le y \le 6$	The outputs: everything that comes out. Each value once.
Reasonable domain and range	the ones that fit the story	Time is not negative, people are whole numbers, a room fills up.
Inequality	$0 \le t \le 8$	t is between 0 and 8, ends included. Words are fine too.
Continuous, discrete	connected piece, separate dots	Inequality for connected; list for separate (Lesson 2).

> THE ONE SENTENCE TO SAY OUT LOUD
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> Domain is the inputs, x, left to right. Range is the outputs, y, bottom to top. Ask what makes sense in the story, then write it in words or as an inequality, with units.

Lesson timeline (40 minutes)

0:00 to 0:05	**Warm-up: two piles.** Write $y = 2x + 1$ and a table with $x = 1, 2, 3, 4$. Ask: "What went in? What came out?" Name the piles: domain (the inputs) and range (the outputs). Then ask: "If there were no table, what could you feed in?" Anything. So the domain of a bare rule is all numbers.
0:05 to 0:14	**Mini-lesson: the story shrinks the piles.** The candle: height $= 8 - t$. Ask the three questions out loud: smallest t (0, when it is lit), largest t (solve $8 - t = 0$, so 8), fractions or whole things (any amount of time). Write $0 \le t \le 8$ and say it in words. Feed in both ends: range $0 \le$ height ≤ 8. Then the van with 7 seats: whole riders only, so the domain is a list. Say: "Continuous gives an inequality, discrete gives a list."
0:14 to 0:24	**Reading a graph.** On the hike graph, run a finger along the bottom from where the graph starts to where it ends: that is the domain. Run a finger up the side from the lowest point to the highest: the range. Do the same on the books graph, where the answer is a list. Then let the student do the three Read these graphs widgets, typing the smallest and largest x and y.
0:24 to 0:35	**Practice.** Interactive practice problems or Worksheet Parts A to C. Same buttons: Hint, Show next step, Reveal answer, Hide help. Insist on units and on the reason for every limit ("why can't t be 9?").
0:35 to 0:40	**Exit ticket.** Worksheet Part E or an index card: (1) the draining pool, domain and range; (2) the described graph, domain and range. Success looks like: $0 \le t \le 20$ and $0 \le$ water ≤ 500; $-2 \le x \le 6$ and $-1 \le y \le 5$.

Misconceptions to listen for

- **Swapping domain and range.** Domain is x, left and right. Range is y, up and down. Say both words with their plain English every time.
- **Ignoring the story.** A bare rule takes any number; the story does not. Ask the three questions before writing anything.
- **Listing an output twice.** If two inputs give 4, the range says 4 once. The range is the pile of values, not a column.
- **Writing an inequality for separate dots.** If the input is people or tickets, the domain is a list of whole numbers.
- **Reading the range from the endpoints only.** The lowest or highest point of a graph can be in the middle. Look at the whole graph.
- **Reaching for interval notation.** Some students have seen brackets and parentheses. Math 1 uses words or inequalities. Keep it there.

Support and stretch

Support

Color the domain along the bottom of the graph and the range up the side with two highlighters. For stories, give the three questions as a printed checklist and have the student answer them in full sentences before writing the inequality.

Stretch

Give a rule with a hidden limit: the area of a square with side s, or the cost of a taxi ride that rounds up to whole minutes. Ask for a story whose domain is a list but whose range is not a simple pattern. Ask what the range of $y = 2x + 1$ is if the domain is $0 \leq x \leq 5$.

Where this goes next

Lesson 4: reading a graph, the behaviors. Domain and range are the first thing to read off any graph; next come the intercepts, where it climbs and falls, where it is above and below the axis, its highest and lowest points, and what happens at the far ends.

> THE HABIT TO BUILD
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> Before writing a domain or range, ask the three questions out loud: smallest input, largest input, whole things or any number? Then say the units.