Domain and Range: The Inputs and the Outputs

Domain is the inputs, the x values a function may take. Range is the outputs, the y values that come out. Find them from a table, from a story, or by reading the edges of a graph.

1 THE VOCABULARY
Words we use

Domain the inputs: every x value the function is allowed to take.

Range the outputs: every y value that comes out. List each value once.

Reasonable domain and range the inputs and outputs that make sense in the story: time starts at 0, people are whole numbers, a room holds only so many.

Inequality $0 \le x \le 8$ means x is between 0 and 8, ends included. Words are fine too.

Continuous one connected piece, so write an inequality.

Discrete separate dots, so write a list or say whole numbers only.

- Watch out for these.
- Mixing them up: domain is x (left and right), range is y (up and down).
- Forgetting the story: a plain rule takes any number, but a candle cannot burn for −2 hours.
- Listing an output twice: if $y = 4$ comes out for two inputs, the range says 4 once.
- Writing an inequality for separate dots: friends come in whole numbers, so list them.

2 THE METHOD
Finding domain and range

1. **FROM A TABLE** domain = the x column, range = the y column with repeats removed.
2. **FROM A STORY** ask about the inputs: smallest that makes sense, largest, fractions or whole things? Then feed in the smallest and largest input to get the range.
3. **FROM A GRAPH** domain: read left to right along the bottom, from where it starts to where it ends. Range: read bottom to top along the side, lowest point to highest.
4. **WRITE IT** words or an inequality ($0 \le x \le 8$) for a connected graph; a list (1, 2, 3, 4) for separate dots. Say the units.

> **WHY A STORY SHRINKS THE DOMAIN**
> The rule $8 - t$ would happily take $t = -2$ or $t = 50$. The candle would not. The reasonable domain keeps only the inputs the story can actually use, and the range follows from feeding those inputs in.

3 A STORY
A candle: height = 8 − t

height = 8 − t	the rule: an 8-inch candle burns 1 inch every hour. t is the input (hours), height is the output (inches)
smallest t = 0	the candle is lit at t = 0. Time before that is not part of the story
largest t: 8 − t = 0, so t = 8	the candle is gone when the height reaches 0. Solve it: t = 8
domain: $0 \le t \le 8$	the inputs that make sense, in hours. Time flows, so every value in between is real (continuous)
t = 0 gives 8, t = 8 gives 0	feed in both ends of the domain to find the ends of the range
range: $0 \le$ height ≤ 8	the outputs that happen, in inches

> **CHECK WITH THE GRAPH**
> Plot (0, 8) and (8, 0) and draw the segment between them. It runs from x = 0 to x = 8 (the domain) and from y = 0 up to y = 8 (the range). The picture and the story agree.

> **SAY IT OUT LOUD**
> Domain is the inputs, x, left to right. Range is the outputs, y, bottom to top. Ask what makes sense in the story, then write it in words or as an inequality.

Unit 2, Lesson 3 in one line: domain is the inputs (x, left to right), range is the outputs (y, bottom to top), and a story keeps only the values that make sense.