Building and Using Exponential Models

Students build exponential models from a story, a table, and two points, use them forward (evaluate) and backward (table, or a horizontal target line and the crossing), interpret a and b, decide discrete versus continuous, and predict what changes if a or b changes.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 7, Lesson 5. Closes Unit 7.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson, pencil, calculator, graph paper
PRIOR KNOWLEDGE	y = a · b^x and the two jobs (Lesson 2). Percent to factor (Lesson 3). Crossing points solve systems (Unit 6, Lesson 1). Discrete versus continuous (Unit 6, Lesson 6).
STANDARDS	NC Math 1 NC.M1.F-LE.5 (interpret parameters in context), NC.M1.F-BF.1 (build a function from a context), NC.M1.A-REI.11 (solutions as intersections), NC.M1.A-CED.1.
KEEP OUT	Solving for x with exponent rules or logarithms (Unit 13). Two points not one step apart. Compound interest formulas (Unit 8).

Learning goals

1. Build y = a · b^x from a story, a table, or two points with one at x = 0, and say what a and b mean.
2. Answer forward questions by evaluating, power first.
3. Answer backward questions by table or by the crossing with a horizontal target line.
4. Decide discrete versus continuous and answer accordingly; predict what changes if a or b changes.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Model	$y = a \cdot b^x$ for a situation	Say what a and b mean.
Forward	given x	Evaluate.
Backward	given y	Table or crossing.
Target line	y = the value	The crossing is the answer.
Discrete, continuous	whole steps or any x	Answer accordingly.

> THE ONE SENTENCE TO SAY OUT LOUD
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> a starts it, b multiplies it. Forward, evaluate. Backward, find the crossing. Say what the numbers mean.

Lesson timeline (45 minutes)

0:00 to 0:05	**Warm-up: the phone.** $800, loses 20% a year. Ask for a and b (watch for 0.20) and the model.
0:05 to 0:17	**Mini-lesson: three ways in, then forward and backward.** Story, table, two points. Forward: 3 years, power first. Backward: build the table to $200; then the target line on the graph and the crossing, tied to Unit 6. Discrete: answer with a whole year.
0:17 to 0:27	**What changes?** 15% instead of 20%: predict, then check on the comparison graph. $1,000 instead of $800: predict, then check. Connect to the family question.
0:27 to 0:40	**Practice.** The practice problems or Worksheet Parts A to E, including the plotted target line. Same buttons: Hint, Show next step, Reveal answer, Hide help. Require the meaning of a and b every time.
0:40 to 0:45	**Exit ticket.** Worksheet Part F. Success looks like $y = 20000 \cdot 0.75^x$ with meanings, $11,250, and the target-line method.

Misconceptions to listen for

- b = 0.20 for losing 20%. Keep 80%: 0.80.
- Multiply a and b first. Power first.
- Bare numbers. a and b must be said in the story's words.
- Year 6.2. A yearly model is discrete: say year 7.
- Changing a changes the shape. a moves the start; b changes the fall or climb.

Support and stretch

Support

Keep the phone table on the desk. Draw the target line in a different color. Let the student use the calculator for powers but say power first aloud each time.

Stretch

Ask when the phone is worth half its price (year 4, about $328 vs $400: between 3 and 4, so during year 4). Ask what b would make it halve in exactly one year.

Where this goes next

Unit 8: compound interest, where the bank multiplies by the same factor every period, compared with simple interest, which adds. Unit 9: geometric sequences, exponential functions on whole-number inputs. Unit 13: the exponent rules.

> THE HABIT TO BUILD
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> Find a and b, say what they mean, evaluate forward, and go backward by the crossing.