Exponential Regression

Students choose between a linear and an exponential model from roughly constant differences or ratios, fit an exponential with technology, interpret a and b in context, answer a when-will-it-reach question, and explain why real data only comes close to the story's model.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 10, Lesson 3.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson (regression tool), pencil, calculator with a power key. Optional: a cup of pennies to run the experiment. A graphing calculator can be used instead of the site's tool.
PRIOR KNOWLEDGE	y = a · b^x, growth and decay factors (Units 7 and 8). Linear or exponential from a table (Unit 7, Lesson 5). Residuals (Lesson 3).
STANDARDS	NC.M1.S-ID.6c, NC.M1.F-LE.1, NC.M1.F-LE.5, NC.M1.F-IF.8b.
KEEP OUT	Proving the model is appropriate. Logarithms. How the tool computes the fit.

Learning goals

1. Decide whether to try a line or an exponential from roughly constant differences or ratios and the shape of the plot.
2. Fit an exponential with technology and write the model.
3. Interpret a and b in the story, including the percent growth or decay.
4. Answer a when-will-it-reach question with a table or graph, and explain why real data wobbles around the story's model.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Exponential regression	best-fitting exponential curve	Technology gives a and b.
Roughly constant	close, not exact	Real ratios wobble.
Growth, decay factor	b	1 plus or minus the rate.
Initial value	a	The start.

> THE ONE SENTENCE TO SAY OUT LOUD
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> Adds or multiplies, even when it is messy. Roughly constant ratios: fit an exponential, and read a and b in the story.

Lesson timeline (45 minutes)

0:00 to 0:05	Warm-up: Lesson 3's surprise. The gnats with a line: r of 0.98 and a U in the residuals. Ask: which family should we have tried?
0:05 to 0:17	Mini-lesson: the gnats. Differences and ratios. Roughly constant. Fit the exponential and switch back to the line to compare. Read a and b. Find 200 gnats with the table and the horizontal line.
0:17 to 0:30	The pennies. If you have pennies, run it. Ask the story for a model first: 300 times one half to the x. Fit: about 348 times 0.49 to the x. Discuss why it is close and not exact. Hand-fit with the explorer.
0:30 to 0:41	Practice. The practice problems or Worksheet Parts A to D. Same buttons: Hint, Show next step, Reveal answer, Hide help.
0:41 to 0:45	Exit ticket. Worksheet Part E. Success looks like about 20 at the start, growing about 25% a day, and the two pieces of evidence.

Misconceptions to listen for

- **The ratios must be exactly equal.** Roughly constant is enough to try.
- **b = 1.63 means 163% growth.** 63%: the factor is 1 plus the rate.
- **a must equal the first data value.** The curve fits all the points.
- **r of 0.98 for a line settles it.** Check the ratios and the residuals.
- **We proved it is exponential.** We supported it.

Support and stretch

Support

Have the student compute ratios with a calculator and round to one decimal so roughly constant is easy to see. Keep the Unit 8 factor card (1 plus or minus the rate) on the desk.

Stretch

Ask for the residuals of the exponential fit on the gnats and whether they still show a pattern. Ask what the model predicts for 10 hours and whether that is believable.

Where this goes next

Lesson 5: association is not causation. Two quantities can move together without one causing the other.

> THE HABIT TO BUILD
>
> Check differences and ratios, fit the family that matches, read a and b in the story, and say about.