Geometric Sequences, Exponentials Made of Dots

Students find the common ratio, write recursive and explicit rules for a geometric sequence, explain the n minus 1, connect the ratio to the growth factor from Unit 7, and interpret a function value in context.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 9, Lesson 2.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson (the explorer needs a screen), pencil, calculator with a power key
PRIOR KNOWLEDGE	Arithmetic sequences and both kinds of rule (Lesson 2). y = a · b^x, growth and decay, the asymptote (Unit 7).
STANDARDS	NC.M1.F-IF.3, NC.M1.F-BF.1a, NC.M1.F-BF.2, NC.M1.F-IF.2.
KEEP OUT	Negative common ratios. Sums of geometric sequences. Solving for n.

Learning goals

1. Find the common ratio, including a fraction, and say whether the terms grow or shrink.
2. Write the recursive rule and the explicit rule, and explain why the exponent is n minus 1.
3. Connect the common ratio to the growth factor b and the terms to dots on an exponential curve.
4. Find a far term, and interpret a function value such as f(3) = 50 in words.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Geometric sequence	multiplies by the same number	Dots on a curve.
Common ratio	r	The growth factor.
Recursive rule	$a_n = r \cdot a_{n-1}$	The stairs.
Explicit rule	$a_1 \cdot r^{n-1}$	The elevator.

> THE ONE SENTENCE TO SAY OUT LOUD
>
> Multiplying gives geometric. The common ratio is the growth factor. Divide to find it, and count n minus 1 multiplications.

Lesson timeline (45 minutes)

0:00 to 0:05	Warm-up: next three terms. 2, 8, 32; then 400, 200, 100. Ask what each is multiplied by, and name r.
0:05 to 0:17	Mini-lesson: two rules. Divide to find r. Grow or shrink by comparing with 1. Recursive, then explicit, with the n minus 1 reason. Nursery year 6: 2,048. Run the stair-step graph.
0:17 to 0:27	It is an exponential. r is b. Dots on the curve. x starting at 0 versus n starting at 1. Slide r across 1 in the explorer.
0:27 to 0:33	A fraction ratio. 400 times one half to the x. Say f(3) = 50 in words. Never zero.
0:33 to 0:41	Practice. The practice problems or Worksheet Parts A to C. Same buttons: Hint, Show next step, Reveal answer, Hide help.
0:41 to 0:45	Exit ticket. Worksheet Part E. Success looks like r = 3, both rules, and 972.

Misconceptions to listen for

- **Subtract to find r.** Divide.
- **The exponent is n.** n – 1: the first term has not been multiplied yet.
- **Multiply first, then the power.** Power first.
- **A shrinking list reaches 0.** It only gets close.
- **f(3) = 50 means 3 times 50.** Input 3, output 50.

Support and stretch

Support

Write the multiplications between the terms as small arrows and count them. Use the explorer with r = 2 and r = one half before other values.

Stretch

Ask for the first term of a geometric sequence whose 2nd term is 12 and 3rd term is 36. Ask which term of 3, 6, 12, 24 first passes 1,000 (the 10th: 1,536).

Where this goes next

Lesson 4: recursive and explicit, both ways. Turn one kind of rule into the other, and see when each is handier.

> THE HABIT TO BUILD
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> Divide to find r, write the start and the rule, power first, and count n minus 1 multiplications.