Geometric Sequences, Exponentials Made of Dots

Geometric: multiply by the common ratio r each time. Recursive: a1 and a_n = r · a_(n-1). Explicit: a_n = a1 · r^(n-1). r is the growth factor b.

1 THE VOCABULARY
Words we use

Geometric sequence multiplies by the same number each time; dots on an exponential curve.

Common ratio r the number multiplied; the growth factor. Term divided by the one before.

Recursive rule a_1 = start; $a_n = r \cdot a_{n-1}$.

Explicit rule $a_n = a_1 \cdot r^{n-1}$.

Grows, shrinks r above 1 grows; between 0 and 1 shrinks, never reaching 0.

- Watch out for these.
- Subtracting to find r. Divide.
- Using r^n instead of r^{n-1}.
- Multiplying a_1 by r before the power. Power first.
- Thinking a shrinking list reaches 0.

2 THE METHOD
Build the rules

1. **FIND** r a term divided by the one before it.
2. **RECURSIVE** a_1 = start; $a_n = r \cdot a_{n-1}$.
3. **EXPLICIT** $a_n = a_1 \cdot r^{n-1}$.
4. **FAR TERM** power first (n – 1 multiplications), then times a_1.
5. **READ A VALUE** f(3) = 50: input 3 in the story, output 50 in the story.

WHY N – 1
The first term has not been multiplied yet. Reaching the nth term takes n – 1 multiplications by r.

3 THE NURSERY
2, 8, 32, 128, ...

$r = 8 \div 2 = 4$	the common ratio, the growth factor
$a_1 = 2$, $a_n = 4 \cdot a_{n-1}$	recursive
$a_n = 2 \cdot 4^{n-1}$	explicit
$a_6 = 2 \cdot 4^5 = 2 \cdot 1024$	power first
= 2,048 trees	year 6

A FRACTION RATIO
400, 200, 100, 50: r = 12. f(3) = 50 means after 3 halvings, 50 is left. Never 0.

SAY IT OUT LOUD
Multiplying gives geometric. The common ratio is the growth factor. Divide to find it.