EDSTAR ANALYTICS | Deep Dive 5
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Geometric Sequences, Exponentials Made of Dots

Geometric: multiply by r each time. Divide to find r. Explicit: a sub n = a sub 1 times r to the (n - 1). Power first.

Name _____ Date _____

Words we use say the math word, then the plain English

WORD	PLAIN ENGLISH	WHAT YOU DO
Geometric sequence	A list that multiplies by the same number each time.	Its dots sit on an exponential curve.
Common ratio, r	The number multiplied each time. It is the growth or decay factor, the b of Unit 7.	Divide any term by the one before. It can be a fraction.
Recursive rule	How to get the next term from the one before. The stairs.	a_1 = start, $a_n = r \cdot a_{n-1}$.
Explicit rule	How to find any term from its term number. The elevator.	$a_n = a_1 \cdot r^{n-1}$.
Grows or shrinks	r bigger than 1 grows; r between 0 and 1 shrinks.	Compare r with 1.
Never zero	A shrinking geometric sequence gets close to 0 but never reaches it.	The x-axis is the asymptote.

Part A Find r and write both rules

SEQUENCE	R	GROWS OR SHRINKS	RECURSIVE RULE	EXPLICIT RULE
3, 6, 12, 24, ...				
2, 8, 32, 128, ...				
5, 15, 45, 135, ...				
80, 40, 20, 10, ...				

Part B Far terms

1. The nursery plants 2, 8, 32, 128, ... trees. How many in year 6? How many multiplications?

2. The 10th term of 3, 6, 12, 24, ...

3. The 6th term of 80, 40, 20, 10, ... Write the ratio as a stacked fraction.

Part C A function value in words

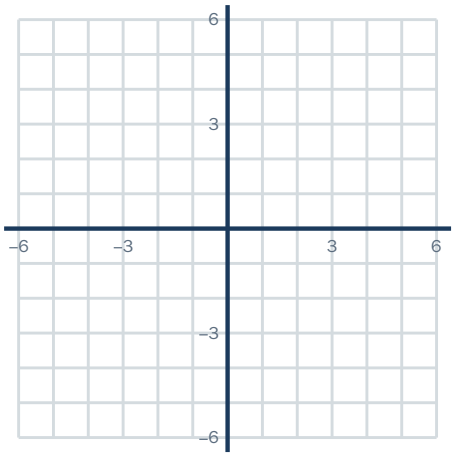
$f(x) = 400 \cdot (\frac{1}{2})^x$ gives the milligrams of medicine left after x hours.

x	0	1	2	3	4
f(x)					

1. What does f(3) = 50 mean, in words?

2. Will the amount ever be exactly 0? Why?

Plot 1, 2, 4, 8 as dots (term number across, term up). Sketch the curve faintly.



Part D The link to Unit 7

1. What is the common ratio called in $y = a \cdot b^x$?

2. Unit 7 used the exponent x. A sequence uses n – 1. Why?

Part E Exit ticket

1. 4, 12, 36, 108, ...: r, both rules, and the 6th term.

2. r = three fourths: grow or shrink? Reach 0?

Answer key

Part A. r = 2, grows; $a_1 = 3$, $a_n = 2 \cdot a_{n-1}$; $a_n = 3 \cdot 2^{n-1}$. r = 4, grows; $a_1 = 2$, times 4; $2 \cdot 4^{n-1}$. r = 3, grows; $a_1 = 5$, times 3; $5 \cdot 3^{n-1}$. r = one half, shrinks; $a_1 = 80$, times one half; $80 \cdot (\frac{1}{2})^{n-1}$.

Part B. 1. 5 multiplications: $2 \cdot 4^5 = 2{,}048$. 2. $3 \cdot 2^9 = 1{,}536$. 3. $80 \cdot (\frac{1}{2})^5 = 2.5$.

Part C. 400, 200, 100, 50, 25. 1. After 3 hours (3 halvings), 50 milligrams are left. 2. No: half of a positive number is still positive. Plot: (1, 1), (2, 2), (3, 4), (4, 8).

Part D. 1. The growth factor, b. 2. A sequence starts counting at n = 1, and the first term has not been multiplied yet, so there are n – 1 multiplications.

Part E. 1. r = 3; $a_1 = 4$, $a_n = 3 \cdot a_{n-1}$; $a_n = 4 \cdot 3^{n-1}$; $a_6 = 972$. 2. Shrinks; never reaches 0.