Growth and Decay

b bigger than 1: growth. b between 0 and 1: decay. b = 1: flat. Percent to factor: start from 100%, add or take away, write as a decimal. Factor to percent: measure from 100%.

1 THE VOCABULARY
Words we use

Exponential growth b bigger than 1: each step multiplies by more than 1.
Exponential decay b between 0 and 1: each step keeps a fraction.
Growth or decay factor the number b multiplied each step.
Growth or decay rate the percent change: how far b is from 1.
b = 1 multiplying by 1 does nothing: a flat line, not exponential.
b must be positive a negative b would flip signs every step.

- Watch out for these.
- Using the percent as the factor: 10% growth is 1.10, not 0.10.
- Forgetting to take away for decay: losing 25% is 0.75, not 1.25 or 0.25.
- Reading b = 0.85 as a 85% loss. It keeps 85% and loses 15%.
- Calling b = 1 growth. It is flat.

2 THE METHOD
Percent and factor

1. GROWTH PERCENT TO FACTOR 100% + the percent, as a decimal. Grows 10%: 1.10.
2. DECAY PERCENT TO FACTOR 100% minus the percent, as a decimal. Loses 25%: 0.75.
3. FACTOR TO RATE above 1: factor minus 100% is the growth rate. Below 1: 100% minus factor is the decay rate.
4. READ THE STORY a is the amount at the start; b says how much remains (or how much more) each step.

> **WHY THE FACTOR LIVES NEXT TO 1**
> Multiplying by 1 is doing nothing. Every growth factor is 1 plus a bit; every decay factor is 1 minus a bit. The bit is the rate.

3 A STORY
$50 \cdot 0.85^x$ milligrams after x hours

a = 50	the starting dose: 50 mg at hour 0
b = 0.85	85% of the medicine remains each hour
100% − 85% = 15%	the decay rate: it loses 15% every hour
decay	b is between 0 and 1
hour 1: 42.5 mg; hour 2: 36.1 mg	multiply by 0.85 each hour

> **GOING THE OTHER WAY**
> A town of 2,000 growing 5% a year: 100% + 5% = 105%, b = 1.05, $y = 2000 \cdot 1.05^x$.

> **SAY IT OUT LOUD**
> Above 1 grows, below 1 decays, 1 does nothing. Start from 100%: add for growth, take away for decay.