How Many Solutions?

Students learn that a system of two lines has one, no, or infinitely many solutions, predict which from m and b, and read the meaning of a false or true final line in the algebra.

TIME	35 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 6, Lessons 1 to 3.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson, pencil, graph paper
PRIOR KNOWLEDGE	Parallel lines and the same line (Unit 4, Lessons 1 and 3). Substitution and elimination (Lessons 2 and 3). Rewriting into y = mx + b.
STANDARDS	NC Math 1 NC.M1.A-REI.6 (solve systems, including recognizing systems with no solution or infinitely many) and A-REI.11.
KEEP OUT	Systems of inequalities (Lesson 5).

Learning goals

1. State the three cases and the picture for each: different slopes cross once; parallel never; the same line everywhere.
2. Predict the number of solutions from m and b before solving, rewriting into y = mx + b first.
3. Interpret a false final line as no solution and a true final line as infinitely many, and explain why the variable disappeared.
4. Write a second equation to make a system have a chosen number of solutions.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
One solution	different m	Cross once.
No solution	same m, different b	Parallel; false line.
Infinitely many	same m and b	The same line; true line.
Predict	compare m and b	Before solving.

> THE ONE SENTENCE TO SAY OUT LOUD
>
> Different slopes cross once. Same slope, different b: parallel, no solution, a false line. Same slope, same b: the same line, infinitely many, a true line. Compare m and b first.

Lesson timeline (35 minutes)

0:00 to 0:05	Warm-up: what can two lines do? Have the student draw two lines three ways: crossing, parallel, on top of each other. Ask how many shared points each has.
0:05 to 0:15	Mini-lesson: predict, then the endings. Compare m and b for the three pairs against $y = 2x + 1$. Then solve each by substitution and watch the ending: x = a number, $1 = -3$, $2 = 2$. Say: the strange line is the answer, not a mistake.
0:15 to 0:23	Predict, then look. The three interactive grids: choose from m and b, then see the picture.
0:23 to 0:32	Practice. The practice problems or Worksheet Parts A to C. Same buttons: Hint, Show next step, Reveal answer, Hide help.
0:32 to 0:35	Exit ticket. Worksheet Part E. Success looks like: none, parallel; infinitely many, the same line.

Misconceptions to listen for

- **0 = 5 means I made a mistake.** It means no solution. Check by comparing m and b.
- **0 = 0 means no solution.** Always true means every point works: infinitely many.
- **Comparing before rewriting.** Standard form hides m and b.
- **Same b means the same line.** Only with the same m too.

Support and stretch

Support

Use the three drawings from the warm-up as a card. Always rewrite both equations under each other in y = mx + b before comparing.

Stretch

Ask for a system in standard form with infinitely many solutions whose two equations look different. Ask how many solutions a system of three lines can have.

Where this goes next

Lesson 5: systems of inequalities. Shade each one the Unit 5 way; the solution is the overlap.

> THE HABIT TO BUILD
>
> Rewrite, compare m, then compare b. Call the number of solutions before solving.