How Many Solutions?

One solution, no solution, or infinitely many. Predict from m and b, confirm by graphing or by the ending of the algebra: a number, a false line, or a true line.

Name _____ Date _____

Words we use

say the math word, then the plain English

WORD	PLAIN ENGLISH	WHAT YOU DO
One solution	The lines cross once: different slopes.	The algebra gives x = a number.
No solution	Parallel lines: same slope, different y-intercept (Unit 4). They never meet.	The algebra ends in a false line like 0 = 5.
Infinitely many solutions	The same line twice: same slope, same y-intercept. Every point on it works.	The algebra ends in an always-true line like 0 = 0.
Predict without solving	Put both in y = mx + b and compare m and b.	Different m: one. Same m, different b: none. Same m and b: infinitely many.
A strange final line	The variables disappear and a number statement is left.	False means none; true means infinitely many. It is not a mistake.

Part A Predict from m and b

Rewrite into y = mx + b when needed. Write one, none, or infinitely many, and the reason.

SYSTEM	M AND B OF EACH	HOW MANY, AND WHY
y = 3x + 2 and y = 3x – 4		
y = 3x + 2 and y = –x + 6		
y = 3x + 2 and 6x – 2y = –4		
2x + 4y = 8 and x + 2y = 6		
y = –2x + 5 and y = 3x – 5		

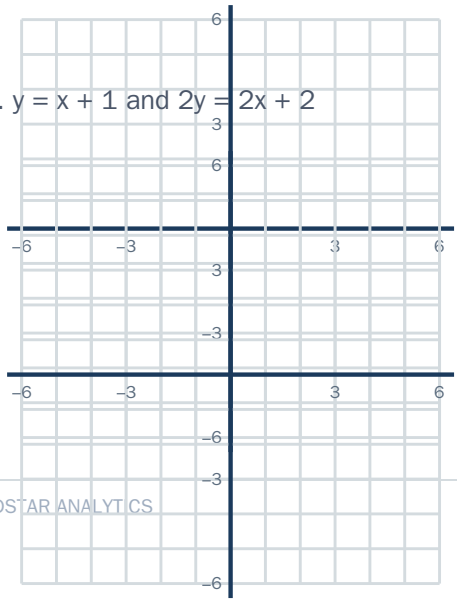
Part B Draw the three pictures

Graph each pair and label it one, none, or infinitely many.

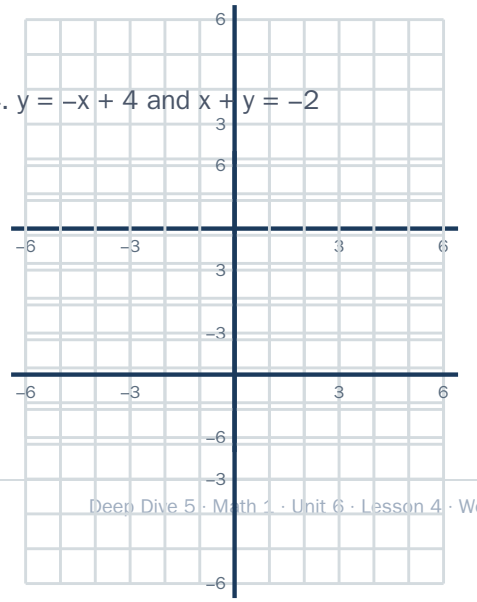
1. y = x + 1 and y = –x + 3

2. y = x + 1 and y = x – 3

3. $y = x + 1$ and $2y = 2x + 2$



4. $y = -x + 4$ and $x + y = -2$



Part C Read the ending

Solve each by substitution or elimination and say what the last line means.

1. $y = 2x + 1$ and $y = 2x - 3$

2. $y = 3x - 2$ and $6x - 2y = 4$

Part D Make it happen, and in your own words

1. The line is $y = 2x + 3$. Write a second equation for no solution, one for infinitely many, and one for exactly one.

2. Why does the variable disappear exactly when the slopes are the same?

3. What is the difference between ending in $0 = 5$ and ending in $0 = 0$?

Part E Exit ticket

1. $y = 4x + 1$ and $y = 4x - 3$: how many, and why?

2. Solving ends in $0 = 0$. What does that mean?

Answer key

Part A. None (same m 3, b 2 and –4). One (m 3 and –1). Infinitely many (6x – 2y = –4 is y = 3x + 2). None (both m = –1/2, b 2 and 3). One (m –2 and 3).

Part B. 1. Cross at (1, 2): one. 2. Parallel: none. 3. One line: infinitely many. 4. Parallel (both slope –1): none.

Part C. 1. 2x + 1 = 2x – 3 gives 1 = –3: false, no solution, parallel. 2. 6x – 2(3x – 2) = 4 gives 4 = 4: true, infinitely many, the same line.

Part D. 1. e.g. y = 2x – 1; 2y = 4x + 6; y = –x + 3. 2. Same slopes make identical x terms that cancel. 3. False: no solution. True: infinitely many.

Part E. 1. None: parallel. 2. Infinitely many: the same line.