Inequalities in the Real World

Students turn a real limit into an inequality (a constraint), graph its feasible region, read which combinations are allowed, treat whole-things constraints as sets of dots, and preview two constraints overlapping for Unit 6.

TIME	40 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 5, Lessons 1 to 3.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson, pencil, graph paper, colored pencils
PRIOR KNOWLEDGE	Graphing a linear inequality (Lessons 1 and 2). Discrete versus continuous (Unit 2, Lesson 2). Rate and starting value in a story (Unit 3, Lesson 5).
STANDARDS	NC Math 1 NC.M1.A-CED.3 (represent constraints by inequalities and interpret solutions as viable or non-viable options in context), NC.M1.A-REI.12.
KEEP OUT	Solving systems (Unit 6). Here only a preview of the overlap.

Learning goals

1. Write a constraint as a linear inequality from a story: variables, total, sign from the words, limit.
2. Graph the feasible region and interpret the boundary (using the limit exactly) and the inside (under the limit).
3. Decide whether a combination is allowed from the picture or from the sentence, and explain why whole-things constraints keep only whole-number points.
4. Check a combination against two constraints and describe the answers as the overlap of two regions.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Constraint	a limit as an inequality	Total, sign, limit.
Feasible	allowed	In the region.
Boundary in a story	the limit exactly	The most you can do.
Discrete, continuous	dots, patch	Whole things versus any amount.
Overlap	obeys both	Unit 6.

> THE ONE SENTENCE TO SAY OUT LOUD
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> A constraint is an inequality: total, sign, limit. Its region is every combination allowed. Whole things: only the dots. Two limits: the overlap.

Lesson timeline (40 minutes)

0:00 to 0:05	Warm-up: can you afford it? Hot dogs $2, drinks $3, $20. Ask: 4 and 3? 7 and 3? 10 and 0? Have the student compute the cost each time. Then ask what rule they were checking. Write it: $2x + 3y \leq 20$.
0:05 to 0:15	Mini-lesson: the feasible region. Graph it with the Lesson 2 procedure. Name the boundary as spending exactly $20 and the inside as under budget. Plot the warm-up points. Add $x \geq 0$ and $y \geq 0$. Then the Unit 2 question: whole hot dogs only, so the answer is the dots.
0:15 to 0:26	Checkers, then two limits. The two interactive checkers with whole-number points marked. Then add the carrying limit and show the overlap on the preview figure. Say: Unit 6.
0:26 to 0:36	Practice. The practice problems or Worksheet Parts A to D. Same buttons: Hint, Show next step, Reveal answer, Hide help. Every allowed-or-not answer needs the sentence.
0:36 to 0:40	Exit ticket. Worksheet Part F. Success looks like $2x + 3y \leq 18$; yes (on the boundary) and no; shade both and take the overlap.

Misconceptions to listen for

- **At most and at least.** At most is ≤; at least is ≥. Read the words.
- **Negative hot dogs.** Add $x \geq 0$ and $y \geq 0$ when the story needs them.
- **Reading fractions off the region.** Whole things snap to whole-number points.
- **Checking one limit when there are two.** Both must hold; the answer is the overlap.
- **The boundary is forbidden.** With ≤ the boundary counts: spending exactly the budget is allowed.

Support and stretch

Support

Start with the warm-up computations before any graph. Use the grid with axes labeled in the story's words. Circle the whole-number points by hand before saying discrete.

Stretch

Ask for the number of whole-number combinations in the budget region. Ask for a constraint where the boundary is dashed (spending less than $20). Ask what a third constraint would do to the overlap.

Where this goes next

Unit 6: systems of equations and inequalities. Two or more constraints at once: shade each, and the solutions are where the regions overlap. The boundary-line work and the shading from this unit carry straight over.

> THE HABIT TO BUILD
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> Total, sign, limit. Shade, then ask: whole things? Then ask: is there a second limit?