Inequalities in the Real World

A constraint is an inequality: the total, the sign, the limit. Its region is every combination allowed. The boundary is the most you can do. Whole things make the answer a set of dots. Two limits overlap.

1 THE VOCABULARY
Words we use

Constraint a limit the answer must obey, written as an inequality.

Feasible allowed: inside the region or on a solid boundary.

Boundary in a story using the limit exactly: the most you can do.

Discrete whole things only: the answer is the whole-number points inside the region.

Continuous any amount: the whole patch is real.

Overlap where two regions both hold: obeys both constraints (Unit 6).

- Watch out for these.
- Writing the limit on the wrong side: at most 20 is ≤ 20, at least 30 is ≥ 30.
- Forgetting $x \geq 0$ and $y \geq 0$ when the story cannot go negative.
- Reading 4.5 hot dogs off the region. Whole things snap to whole-number points.
- Checking only one constraint when there are two.

2 THE METHOD
From story to answer

1. **NAME THE VARIABLES** x and y are the two things you choose (hot dogs and drinks).
2. **WRITE THE TOTAL AND THE LIMIT** cost or amount on the left, the sign from the words (at most $\leq$, at least $\geq$), the limit on the right.
3. **GRAPH IT (LESSON 2)** solve for y, boundary from m and b, solid or dashed, shade, test (0, 0). Keep $x \geq 0$ and $y \geq 0$ if the story needs it.
4. **READ ANSWERS** a combination is allowed if its point is in the region, or if the numbers make the sentence true.
5. **ASK: WHOLE THINGS?** if so, only the whole-number points count. If two limits: shade both, answers in the overlap.

WHY THE BOUNDARY IS THE MOST

On the boundary the inequality is an equation: the cost equals the budget exactly. Every point inside spends less; every point outside spends more.

3 A BUDGET
Hot dogs $2, drinks $3, at most $20

$2x + 3y \leq 20$	x hot dogs, y drinks; cost at most 20
$y \leq -2/3\,x + 20/3$	solve for y (multiply by 1/3, positive, no flip): solid, shade below
test (0, 0): $0 \leq 20$	true: (0, 0)'s side, the corner near the origin
(4, 3): $8 + 9 = 17 \leq 20$	allowed, feasible
(7, 3): $14 + 9 = 23$	not allowed, outside
whole hot dogs and drinks	only the whole-number points inside count: a set of dots

WITH A SECOND LIMIT

Add $x + y \leq 8$ (carrying limit). Shade it too. (5, 2) obeys both; (2, 6) breaks the budget. The answers are in the overlap: Unit 6.

SAY IT OUT LOUD

Total, sign, limit. Region is every combination allowed. Whole things: dots. Two limits: overlap.

Unit 5, Lesson 4 in one line: a constraint is an inequality whose region is every allowed combination; whole-things constraints keep only the dots, and two constraints keep only the overlap.