Solving Inequalities: Gator Eats Greater

Same three steps as an equation. The sign is a gator that eats the bigger number. Multiply both sides by a negative and the gator turns around.

1 THE VOCABULARY
Words we use

Gator eats greater the open side of < or > faces the bigger number: 7 > 3 and 3 < 7.

Or equal to the bar under ≤ and ≥: the boundary number counts too. Closed circle on the number line; open circle without the bar.

Solution every number that makes the sentence true, usually a whole side of the number line: x < 5.

Addition property of inequality add the same thing to both sides, and the sign stays. Adding a negative does NOT flip.

Multiplication property of inequality multiply both sides by the same positive number, sign stays. By the same negative number, the sign turns around.

Turning it around 3 > x and x < 3 are one sentence read from each end. Not a flip: the gator still eats the 3.

Coefficient the number on the variable. Its sign decides the flip: (−2)x needs the reciprocal −12, which is negative.

- Watch out for these four.
- Forgetting the flip: −2x < 8 gives x > −4, not x < −4. Test x = 0 and the check catches it.
- Flipping when you add: 5 + (−7) on both sides changes nothing about which side is bigger.
- Flipping because the other side is negative: 3x ≤ −12 gives x ≤ −4. Only the multiplier's sign matters.
- Reading 3 > x as x > 3. The gator was eating the 3; keep it that way: x < 3.

2 THE METHOD
Three steps, one rule added

1. **REWRITE, so the line is all plus and times** Subtracting is adding the additive inverse (the opposite): 7 - 2x becomes 7 + (−2)x. A written ÷ sign is multiplying by the multiplicative inverse (the reciprocal). Write the invisible 1s in: −x is (−1)x.
2. **DISTRIBUTE, THEN COMBINE THE LIKE TERMS** The distributive property (hand one out to everyone), then the distributive property run backwards (factoring, pulling out what the terms share): 5x + (−2x) = (5 + (−2))x = 3x.
3. **x TERMS ONE SIDE, PLAIN NUMBERS THE OTHER, then one last multiplication** Add opposites to both sides (addition property of inequality, sign stays). Move the x term with the smaller coefficient (the number on the variable) so the one you keep is positive. Last, multiply both sides by the reciprocal of the coefficient. Negative reciprocal: flip the sign.
4. **CHECK THREE NUMBERS** one inside, the boundary, one outside. Written with plus the opposite, never with a minus or ÷.

> **WHY THE FLIP**
> Multiplying by −1 takes opposites, and opposites mirror the number line. x = 2 is left of 5, so x < 5. Its opposite −2 is right of −5, so −x > −5. Any negative number is a positive number times −1, so every negative multiplier mirrors the line.

3 EVERY LINE NAMED
Worked example

7 - 2x ≥ 1	the original: 7 minus 2x is greater than or equal to 1
7 + (−2)x ≥ 1	Step 1: subtracting 2x is adding its additive inverse (the opposite). The coefficient (the number on the variable) is −2
7 + (−2)x + (−7) ≥ 1 + (−7)	Step 3: the addition property of inequality (add the same thing to both sides), adding the additive inverse (the opposite) of 7. The sign stays
(−2)x ≥ −6	7 + (−7) = 0, and the additive identity (the do-nothing number 0) leaves (−2)x. On the right, 1 + (−7) = −6
(−12) × (−2)x ≤ (−12) × (−6)	the multiplication property of inequality, by the multiplicative inverse (the reciprocal) of −2. It is negative, so the sign turns around: ≥ becomes ≤
1x ≤ 3, so x ≤ 3	(−12) × (−2) = 1, and 1x = x by the multiplicative identity (the do-nothing number 1). On the right, (−12) × (−6) = 3

> **CHECK THREE NUMBERS**
> x = 0: 7 + (−2) × 0 = 7, and 7 ≥ 1 is true. x = 3, the boundary: 7 + (−6) = 1, and 1 ≥ 1 is true, so 3 is included. x = 5: 7 + (−10) = −3, and −3 ≥ 1 is false. A forgotten flip (x ≥ 3) would have claimed x = 5 works.

> **SAY IT OUT LOUD**
> Rewrite into plus and times, add the opposite, multiply by the reciprocal, and if that reciprocal is negative, the gator turns around.

Lesson 6 in one line: same steps, and a negative multiplier flips the sign.