Linear or Exponential?

Students learn the table test (equal x steps, subtract for a constant difference, divide for a constant ratio, else neither), read story clue words, and see through Town A versus Town B and the doubling penny why exponential growth eventually exceeds any line.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 7, Lesson 4.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson, pencil, calculator for the race table
PRIOR KNOWLEDGE	Lines add, exponentials multiply (Lesson 1). Slope from a table (Unit 3). Percent to factor (Lesson 3).
STANDARDS	NC Math 1 NC.M1.F-LE.1 (distinguish linear from exponential from tables, graphs, and descriptions), NC.M1.F-LE.3 (exponential growth eventually exceeds linear).
KEEP OUT	Quadratic tables beyond naming them neither. Compound interest formulas (Unit 8).

Learning goals

1. Run the table test in order: equal steps, subtract, divide, neither, and name the constant.
2. Sort stories by clue words, including the by 10 versus by 10% trap.
3. Compute both towns for several years and explain why the exponential must eventually pass the line.
4. Read a race graph and locate the takeover.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Equal steps	x column	Check first.
Constant difference	subtract	Linear; the slope.
Constant ratio	divide	Exponential; b.
Clue words	per, each, every; doubles, percent	Adds; multiplies.
Eventually exceeds	the race	The exponential's step keeps growing.

> THE ONE SENTENCE TO SAY OUT LOUD
>
> Equal steps. Subtract: same difference, linear. Divide: same ratio, exponential. In the end, the exponential wins.

Lesson timeline (45 minutes)

0:00 to 0:05	Warm-up: two tables. 7, 10, 13, 16 and 7, 21, 63, 189. Ask which adds and which multiplies, and how they know.
0:05 to 0:15	Mini-lesson: the test in order. Equal steps, subtract, divide, neither. Do the 200, 220, 242 table to show why you must divide before saying neither. Then the clue words and the by 10 versus by 10% trap.
0:15 to 0:27	The race. Town A and Town B: write the equations, fill a few years, read the graph, find the takeover. Then the doubling penny with the step table. Say: the exponential eventually exceeds any line.
0:27 to 0:40	Practice. The six table checkers and the story sort, then the practice problems or Worksheet Parts A to D. Same buttons: Hint, Show next step, Reveal answer, Hide help.
0:40 to 0:45	Exit ticket. Worksheet Part F. Success looks like linear with difference 3, exponential with ratio 2, and the by 50 versus by 50% distinction.

Misconceptions to listen for

- **Differences change, so it is neither.** Divide first.
- **Running the test with unequal x steps.** Check the x column.
- **By 10% adds 10.** It multiplies by 1.10.
- **The line is ahead, so it wins.** Look further right; the exponential always catches up.
- **A decreasing table cannot be exponential.** 64, 32, 16 has ratio one half.

Support and stretch

Support

Provide a template with columns for differences and ratios. Use whole-number tables first. Keep the race table on the desk while reading the graph.

Stretch

Ask for the first year Town B is at least 100 people ahead of Town A. Ask whether a line with slope 100 would still lose to Town B, and when.

Where this goes next

Lesson 6: building and using exponential models from stories, tables, and two points, and reading them forward and backward.

> THE HABIT TO BUILD
>
> Equal steps, subtract, then divide. Say the family and its constant.