Linear or Exponential?

Equal x steps first. Subtract: constant difference is linear. Divide: constant ratio is exponential. Neither constant: neither. Stories: fixed amounts add, doubles and percents multiply. Exponential growth eventually exceeds any line.

1 THE VOCABULARY
Words we use

Equal steps x goes up by the same amount each row; check first.

Constant difference subtracting gives the same number: linear, and it is the slope.

Constant ratio dividing gives the same number: exponential, and it is b.

Clue words per, each, every with a fixed amount: linear. Doubles, halves, a percent: exponential.

Eventually exceeds the exponential's step keeps growing; the line's does not.

- **Watch out for these.**
- Stopping after differences change. Divide before saying neither.
- Running the test on unequal x steps.
- By 10 versus by 10%: one adds, one multiplies.
- Thinking the line wins because it is ahead early. Look far enough right.

2 THE METHOD
The table test

1. **EQUAL STEPS?** x must go up by the same amount each row.
2. **SUBTRACT** each y minus the one before. Same every time: linear.
3. **DIVIDE** each y over the one before. Same every time: exponential.
4. **NEITHER** both change: not these families.
5. **NAME THE CONSTANT** the difference is the slope; the ratio is b.

WHY THE EXPONENTIAL ALWAYS WINS
A line adds the same amount every step forever. An exponential adds a percent of a growing total, so its step grows without limit and eventually passes the line's fixed step. After that the gap only widens.

3 THE RACE
Town A adds 10; Town B grows 10%

A: $y = 145 + 10x$	linear: adds 10 each year
B: $y = 145 \cdot 1.10^x$	exponential: multiplies by 1.10 each year
year 1: A 155, B 159.5	close
year 8: A 225, B 311	B pulling away
year 20: A 345, B 975	the gap keeps growing

A TABLE WITH A HIDDEN PERCENT
200, 220, 242, 266.2: differences 20, 22, 24.2 change; ratios all 1.1. Exponential, 10% growth.

SAY IT OUT LOUD
Equal steps. Subtract: same difference, linear. Divide: same ratio, exponential. The exponential wins in the end.

Unit 7, Lesson 5 in one line: with equal x steps, a constant difference means linear and a constant ratio means exponential; fixed amounts add, percents multiply, and exponential growth eventually exceeds any line.