Longer Equations: Combine, Then Undo

1 THE VOCABULARY
Words we use

Like terms terms with the same letter part: 5x and 1x are, 5x and 5 are not.

Coefficient the number on the variable: 3x is 3 × x, and a lonely x is 1x. Write the 1 in.

Constant a plain number with no letter, like the −4 in 6x + (−4).

Distributive property run backwards factoring, pulling out what the terms share, from Lesson 2. That is all combining like terms (terms with the same letter part) is.

Addition property of equality add the same thing to both sides. Lesson 4 called this doing the same to both sides.

Multiplication property of equality multiply both sides by the same thing, the other half of that Lesson 4 rule.

Associative property of multiplication the factors change partners, the grouping moves: 14 × 4x becomes (14 × 4)x.

The opposite rule the opposite of x is (−1)x, so −x is really (−1)x. Not the multiplicative identity (the do-nothing number 1).

- **Watch out for these four.**
- The invisible 1: a lonely x is 1x, and −x is (−1)x (the opposite rule), so 5x - x is 4x, not 5.
- -(x + 4) means (−1) × (x + 4), so the distributive property (hand one out to everyone) gives (−1)x + (−4).
- 4x + 5 is not 9x: the distributive property run backwards (factoring, pulling out what the terms share) needs a shared letter part.
- Nothing crosses the equals sign by itself: add its additive inverse (the opposite) to both sides, or the balance tips.

2 THE METHOD
The three steps

1. **REWRITE, so the line is all plus and times** Subtracting is adding the additive inverse (the opposite): 6x - 4 becomes 6x + (−4). Dividing is multiplying by the multiplicative inverse (the reciprocal): 12 ÷ 3 becomes 12 × 13. Lesson 3 called this translating. Write the invisible 1s in: a lonely x is 1x by the multiplicative identity (the do-nothing number 1), and −x is (−1)x (the opposite rule). Only a written ÷ sign is a division that Step 1 rewrites. A fraction bar is not a written ÷ sign, so Step 1 has nothing to rewrite there. x3 already is the coefficient (the number on the variable) 13 on x. Why first? The commutative property of addition (the terms trade places) and the associative property of addition (the terms change partners, the grouping moves) are written for addition. Once the line is only plus and times, you never have to stop and wonder whether a move is allowed.
2. **DISTRIBUTE, THEN COMBINE THE LIKE TERMS (the terms with the same letter part)** Parentheses first: the distributive property (hand one out to everyone) gives 3(x - 4) = 3x + (−12). Distributing over a subtraction is allowed, as Lesson 2 showed. Then slide them together with the commutative property of addition (the terms trade places), and group them with the associative property of addition (the terms change partners, the grouping moves). Combine them with the distributive property run backwards (factoring, pulling out what the terms share): 5x + 1x = (5 + 1)x = 6x. Two plain numbers share no letter, so you just add them.
3. **x TERMS ONE SIDE, PLAIN NUMBERS THE OTHER, then one last multiplication** Move the x term with the smaller coefficient (the number on the variable). Then the x you keep has a positive coefficient (the number on the variable), and the numbers stay easy. Add that term's additive inverse (the opposite) to both sides, by the addition property of equality (add the same thing to both sides). Do the same with a plain number. Last, multiply both sides by the multiplicative inverse (the reciprocal) of the coefficient (the number on the variable). That is the multiplication property of equality (multiply both sides by the same thing). These are Lesson 1's undo partners.

3 EVERY LINE NAMED
Worked example

5x - 4 + x = 2x - 12	the original
5x + (−4) + 1x = 2x + (−12)	Step 1: subtracting is adding the additive inverse (the opposite). The lonely x gets its 1 from the multiplicative identity (the do-nothing number 1)
(5 + 1)x + (−4) = 2x + (−12)	Step 2: the commutative property of addition (the terms trade places) slides, the associative property of addition (the terms change partners, the grouping moves) groups, the distributive property run backwards (factoring, pulling out what the terms share) combines
6x + (−4) = 2x + (−12)	5 + 1 = 6
6x + (−4) + (−2x) = 2x + (−12) + (−2x)	Step 3: the addition property of equality (add the same thing to both sides), adding the additive inverse (the opposite) of 2x, the smaller x term
(6 + (−2))x + (−4) = (2 + (−2))x + (−12)	the same slide and group on each side, then the distributive property run backwards (factoring, pulling out what the terms share)
4x + (−4) = −12	6 + (−2) = 4. On the right, 0x = 0 by the zero rule (0 times anything is 0), and the additive identity (the do-nothing number 0) leaves −12
4x + (−4) + 4 = (−12) + 4	again the addition property of equality (add the same thing to both sides), with the additive inverse (the opposite) of −4
4x = −8	(−4) + 4 = 0, and the additive identity (the do-nothing number 0) leaves 4x alone. On the right, (−12) + 4 = −8
14 × 4x = 14 × (−8)	the multiplication property of equality (multiply both sides by the same thing), by the multiplicative inverse (the reciprocal) of the coefficient (the number on the variable) 4
(14 × 4)x = −2, so 1x = −2 and x = −2	the associative property of multiplication (the factors change partners, the grouping moves). Then 14 × 4 = 1, and 1x = x by the multiplicative identity (the do-nothing number 1). On the right, 14 × (−8) = −2

CHECK
5 × (−2) + (−4) + 1 × (−2) = (−10) + (−4) + (−2) = −16, and 2 × (−2) + (−12) = −16. Both sides are −16, so x = −2 is right.

SAY IT OUT LOUD
Rewrite into plus and times, combine the like terms (the terms with the same letter part), then add the opposite and multiply by the reciprocal.

Lesson 5 in one line: rewrite, combine, then undo.