EDSTAR ANALYTICS | Deep Dive 5
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Algebra, One Idea at a Time

Say the math word, then the plain English, every time. Each lesson builds on the one before it: the do-nothing numbers and undo partners (1), the properties that only addition and multiplication pass (2), so you translate subtraction and division away first (3), and then solving is just undoing (4).

1 IDENTITY AND INVERSE
The Do-Nothing Numbers and Their Undo Partners

Additive identity the do-nothing number for adding: **a + 0 = a**

Multiplicative identity the do-nothing number for multiplying: **a × 1 = a**

Additive inverse the **opposite**, the undo partner for adding: **a + (−a) = 0**

Multiplicative inverse the **reciprocal**, the undo partner for multiplying: **a × 1/a = 1**

- Opposite of 6 is −6; of −2.5 is 2.5. Reciprocal of 2/5 is 5/2; of −4 is −1/4 (the sign stays). 0 has no reciprocal: nothing times 0 makes 1.
- **The hunt for hidden 1s and 0s:** 3/4 = 9/12 (you multiplied by 3/3, which is 1). 2 ft × 12 in/1 ft = 24 in (that fraction is 1). x + 5 = 12: add −5 to both sides and a 0 appears, so x = 7.

> **SAY IT OUT LOUD**
> An inverse is defined by the identity. The opposite brings a number back to 0. The reciprocal brings it back to 1.

2 THE PROPERTIES
Commute, Associate, Distribute

Commutative to commute is to travel back and forth, so the numbers **trade places**: **a + b = b + a**, **a × b = b × a**

Associative an associate is a partner, so you may **change partners** (move the parentheses): **(a + b) + c = a + (b + c)**

Distributive to distribute is to **hand one out to everyone** inside: **a × (b + c) = a × b + a × c**

DOES IT...	ADD	SUBTRACT	MULTIPLY	DIVIDE
commute? (trade places)	yes	no	yes	no
associate? (change partners)	yes	no	yes	no

- Proof by numbers: 10 − 4 = 6 but 4 − 10 = −6. (24 ÷ 6) ÷ 2 = 2 but 24 ÷ (6 ÷ 2) = 8.
- Watch the direction: 7 × (10 + 3) = 70 + 21 = 91, but 7 + (10 × 3) is not (7 + 10) × (7 + 3).

> **THE BIG IDEA**
> Only addition and multiplication pass every test. So algebra keeps those two and puts the other two in disguise (Lesson 3).

3 STEP ONE
Translate Subtraction and Division

Subtraction → addition **a − b = a + (−b)**. Change − to +, swap in the **opposite**. "Subtract 5" becomes "add negative 5."

Division → multiplication **a ÷ b = a × 1/b**. Change ÷ to ×, swap in the **reciprocal**. "Divide by 2" becomes "multiply by one-half."

- **Multiplying fractions** is the friendly one: tops times tops, bottoms times bottoms, then simplify. 4/5 × 3/8 = 12/40 = 3/10. A whole number rides over 1 (6 = 6/1). A number times its reciprocal is 1: 2/3 × 3/2 = 1.
- **Negatives: think owe and have.** −3 + (−4): owe 3 and owe 4, so −7. 5 + (−2): have 5, owe 2, so 3. −8 + 3: still owe 5, so −5. Same signs: add the amounts, keep the sign. Different signs: take the difference, keep the sign of the bigger pile.
- **The invisible 1s.** x means 1x, so x + 2x = 3x and 5x − x = 4x (not 5). x means x^1, so $x \cdot x^2 = x^3$. And −x is (−1)x.

> **SAY IT OUT LOUD**
> Change the operation to its partner and replace the number with its inverse. Minus becomes plus the opposite. Divide becomes times the reciprocal. Then write in the invisible 1s.

4 SOLVING EQUATIONS
Undo to Get the Letter Alone

Equation a balance: the two sides are equal. Whatever you do to one side, do to the other.

Solve get the letter alone: the last line reads **x = the answer**. After Step 1, only a number **added** to x and its **coefficient** (the number on the variable; a lonely x is 1x) can be on x.

1. **Rewrite:** no subtraction, no division, every invisible 1 written in. Now it is all + and ×.
2. **Add the opposite** of the number added to x, on both sides. A 0 appears.
3. **Multiply by the reciprocal** of the coefficient on x, on both sides. A 1 appears.
4. **Check** by putting the answer back in, using plus the opposite.

2x − 6 = 8	the original
2x + (−6) = 8	Step 1 rewrite: subtracting 6 is adding −6
2x + (−6) + 6 = 8 + 6	add the opposite of −6 to both sides; (−6) + 6 = 0
2x = 14	the 0 does nothing, so 2x is alone
½ × 2x = ½ × 14	multiply by the reciprocal of the coefficient, 2; ½ × 2 = 1
x = 7	check: 2 × 7 + (−6) = 14 + (−6) = 8 ✓

> **SOCKS AND SHOES**
> Shoes come off first: add the opposite, then multiply by the reciprocal. x/3 is not a division; it is the coefficient ⅓ on x. Parentheses? Distribute first (Lesson 2).

The four sentences: Identity does nothing, inverse undoes. Commute, associate, distribute: only + and × pass. Rewrite first, reveal the 1s. Add the opposite, multiply by the reciprocal, check.