Perpendicular Lines and the Negative Reciprocal

Students see where the perpendicular slope rule comes from (a quarter turn trades the rise and the run and reverses the lean), learn the negative reciprocal and the product check, handle the horizontal and vertical pair by sight, and write a perpendicular line through a point.

TIME	40 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 4, Lesson 1 (parallel lines).
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson (the explorer needs a screen), pencil, graph paper, a sheet of paper for its square corner, an index card to rotate
PRIOR KNOWLEDGE	Slope as rise over run; the reciprocal as the multiplicative inverse (Unit 1); y = mx + b and put-the-point-in (Unit 3); parallel lines (Lesson 1).
STANDARDS	NC Math 1 NC.M1.G-GPE.5 (use slope to determine whether lines are perpendicular; write the equation of a line perpendicular to a given line through a point).
KEEP OUT	Proving the negative reciprocal rule formally. The quarter-turn picture is the level for Math 1. Distance and midpoint are Unit 11.

Learning goals

1. Explain, with the quarter-turn picture, why perpendicular slopes are negative reciprocals: the rise and the run trade places and the lean reverses.
2. Find the negative reciprocal of any slope (flip, change the sign), including whole numbers, and check with the product being -1.
3. Decide whether two lines are perpendicular from equations and graphs, and handle the horizontal and vertical pair by sight.
4. Write the equation of a line perpendicular to a given line through a given point.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Perpendicular	square corner	Cross at a right angle.
Reciprocal	flip the fraction	The multiplicative inverse from Unit 1.
Negative reciprocal	flip and change the sign	The perpendicular slope.
Product is –1	$m1 \times m2 = -1$	The quick check.
Special pair	$y = c$ and $x = c$	Perpendicular by sight; 0 cannot be flipped.

> THE ONE SENTENCE TO SAY OUT LOUD
>
> Turn it a quarter turn and the rise and the run trade places. Flip the fraction, change the sign. Multiply the slopes and check for negative one.

Lesson timeline (40 minutes)

0:00 to 0:05	Warm-up: turn the card. Draw a staircase on an index card: run 3 to the right, rise 2 up, and the line through it. Have the student turn the card a quarter turn against the corner of a page and read the new staircase: which way is the old run pointing now? Which way is the old rise?
0:05 to 0:15	Mini-lesson: build the rule, then name it. Write the two slopes from the card: 2/3 and –3/2. Ask what happened to the fraction (flipped) and to the sign (changed). Name it: the negative reciprocal. Do the product check together. Then more pairs, including a whole number (4 and –1/4). Then the special pair: y = 4 and x = 3, and why you cannot flip 0.
0:15 to 0:23	Explorer. The student drives the slope slider and watches the perpendicular partner turn with it, reading both slopes off the note under the grid. Ask for m = 2, then 1/2, then –3, then 0.
0:23 to 0:35	The skill, then practice. Perpendicular to y = 2x + 1 through (4, 3): new m, put the point in, solve for b, write, check the product. Then the practice problems or Worksheet Parts A to D. Same buttons: Hint, Show next step, Reveal answer, Hide help.
0:35 to 0:40	Exit ticket. Worksheet Part F or an index card. Success looks like: 5/2 with the check $-2/5 \times 5/2 = -1$; $y = -1/3\,x + 5$.

Misconceptions to listen for

- **Flip but keep the sign.** 2/3 and 3/2 multiply to 1. The quarter turn also reverses the lean, so the sign must change.
- **Change the sign but do not flip.** 2/3 and –2/3 are a reflection, not a quarter turn.
- **A whole number has no reciprocal.** 4 is 4/1; its reciprocal is 1/4.
- **Flipping 0.** There is no 1/0. Horizontal and vertical are perpendicular by sight.
- **Keeping the old m for the perpendicular line.** The new m is the negative reciprocal; only then find b.
- **Perpendicular means opposite slopes.** 3 and –3 are not perpendicular (product –9). Flip and change the sign.

Support and stretch

Support

Do the quarter turn physically with the index card several times before the rule. Keep slopes as fractions with small numbers. Use the sheet-of-paper corner to check every drawn pair.

Stretch

Ask why the product check works (reciprocal times the number is 1; one sign change makes –1). Ask for the perpendicular line through a point when the given line is in standard form. Ask what happens to the perpendicular partner as m gets very large (it flattens toward slope 0) and very small (it steepens).

Where this goes next

Lesson 3: same, parallel, or perpendicular? Both slope jobs together, sorting any pair of lines by their slopes, and choosing the right move to write a parallel or a perpendicular line.

> THE HABIT TO BUILD
>
> For a perpendicular slope: flip the fraction, change the sign, multiply to check for negative one.