Perpendicular Lines and the Negative Reciprocal

Perpendicular lines cross at a right angle. A quarter turn trades the rise and the run and reverses the lean, so perpendicular slopes are negative reciprocals: flip the fraction, change the sign. Product: -1.

1 THE VOCABULARY

Words we use

Perpendicular cross at a right angle, a square corner (90 degrees).

Reciprocal flip the fraction, top and bottom swap: the multiplicative inverse from Unit 1.

Negative reciprocal flip the fraction and change the sign: the perpendicular slope.

Product is –1 the quick check: perpendicular slopes multiply to –1.

The special pair horizontal (slope 0) and vertical (no slope): perpendicular by sight, since 0 cannot be flipped.

- Watch out for these.
- Flipping without changing the sign: 2/3 and 3/2 multiply to 1, not –1. Not perpendicular.
- Changing the sign without flipping: 2/3 and –2/3 are not perpendicular.
- Forgetting a whole number is a fraction: 4 is 4/1, so its negative reciprocal is –1/4.
- Trying to flip 0. The horizontal and vertical pair is handled by sight.
- Keeping the old m when writing the perpendicular line. The new m is the negative reciprocal.

2 THE METHOD

Why, then how

1. **WHY: THE QUARTER TURN** a line with run 3 and rise 2 turned 90 degrees has run 2 and rise 3, leaning the other way. 2/3 becomes –3/2.
2. **THE RULE** negative reciprocal: flip the fraction (reciprocal), change the sign.
3. **THE CHECK** multiply the two slopes. –1 means perpendicular.
4. **A PERPENDICULAR LINE THROUGH A POINT** new m = negative reciprocal of the given m; put the point in; solve for b; write y = mx + b.
5. **THE SPECIAL PAIR** y = c (flat) and x = c (vertical) are perpendicular by sight.

> **WHY THE PRODUCT IS –1**
> A fraction times its reciprocal is 1 (Unit 1: the multiplicative inverse). Changing one sign makes the product –1. So negative reciprocal and product –1 are the same statement, and either one tests for a square corner.

3 PERPENDICULAR THROUGH A POINT

Perpendicular to y = 2x + 1 through (4, 3)

m = 2 = 2/1	the given slope, written as a fraction
flip: 1/2; change the sign: –1/2	the negative reciprocal. This is the new m
3 = –1/2 × 4 + b	put the point (4, 3) into y = mx + b with the new m
3 = –2 + b, so b = 5	multiply, then add 2 to both sides
y = –1/2 x + 5	the perpendicular line
check: 2 × (–1/2) = –1	the product test passes; and 4 makes it true: –2 + 5 = 3

> **ON THE GRID**
> Draw both lines. Where they cross, hold the corner of a sheet of paper against the angle: it fits. A square corner.

> **SAY IT OUT LOUD**
> Quarter turn: rise and run trade places. Flip the fraction, change the sign. Multiply to check for negative one.