Commute, Associate, Distribute: Why the Properties Are Named That

A 25-minute lesson for students about 12 years old. It teaches the commutative, associative, and distributive properties by starting from the everyday meanings of the words, then shows that only addition and multiplication have these properties, which is why algebra treats them as its two operations.

Time	25 minutes (one student with a parent or tutor, or a small group)
Who	Math 1 (algebra readiness), about age 12. Follows Lesson 1 (identity and inverse).
Materials	Student worksheet (PDF), the Deep Dive 5 interactive practice page for this lesson, pencil, whiteboard or scrap paper
Prior knowledge	Lesson 1: the do-nothing numbers (0 and 1) and the undo partners (opposite and reciprocal); subtraction is adding the opposite, division is multiplying by the reciprocal
Standards	Builds toward NC Math 1 NC.M1.A-SSE.1 and A-REI.1 (interpret expressions; justify each step of solving) and rests on grade 6 and 7 work applying properties of operations to generate equivalent expressions (6.EE.3, 7.EE.1).

Learning goals

By the end of the lesson the student can:

- **1.** Explain the **commutative**, **associative**, and **distributive** properties in plain English, using the everyday meaning of each word (commute = travel back and forth, associate = partner up, distribute = hand out to everyone).
- **2.** Test a property with numbers and show, with a counterexample, that subtraction and division do not commute or associate.
- **3.** State the big idea: addition and multiplication are the two operations of algebra because they are the ones with these properties, and subtraction and division are rewritten as adding the opposite and multiplying by the reciprocal.

Vocabulary: the everyday word first, then the math

The rule for the lesson: never say a property's name without saying what the ordinary word means. The names are not random; each one describes exactly what the numbers are allowed to do.

Property	The everyday word	What the numbers do	The rule
Commutative property (of addition, of multiplication)	To **commute** is to travel back and forth between two places, like a parent commuting to work and home again.	The two numbers swap places (commute) and the answer stays the same.	$a + b = b + a$ $a \times b = b \times a$
Associative property (of addition, of multiplication)	An **associate** is a partner. To associate with someone is to pair up or hang out with them.	You can change which numbers are partnered up first (the parentheses) and the answer stays the same.	$(a + b) + c = a + (b + c)$ $(a \times b) \times c = a \times (b \times c)$
Distributive property (of multiplication over addition)	To **distribute** is to hand something out to everyone, like distributing papers to every desk in the room.	The number outside the parentheses gets handed out to every term inside. Every term gets multiplied.	$a \times (b + c) = a \times b + a \times c$
Identity property (from Lesson 1)	Your **identity** is who you are.	Add 0 or multiply by 1 and the number keeps its identity.	$a + 0 = a$ $a \times 1 = a$
Inverse property (from Lesson 1)	**Inverse** means turned around, reversed.	The undo partner brings a number back to the do-nothing number.	$a + (-a) = 0$ $a \times \frac{1}{a} = 1$

The one sentence to say out loud

Addition and multiplication are the two operations of algebra. They are the only ones that commute, associate, and (for multiplication) distribute. Subtraction and division do not, so algebra puts them in disguise: subtracting is adding the opposite, and dividing is multiplying by the reciprocal. Once they are in disguise, every property works again.

Lesson timeline (25 minutes)

- **0:00 to 0:03 Warm-up: three ordinary words**

  Before any math, ask the student to write one sentence for each: "What does it mean when a grown-up **commutes** to work?" "What is a business **associate**?" "What does a teacher do when she **distributes** the worksheets?"

  Expected: commute = go back and forth between two places; associate = a partner, someone you work or pair up with; distribute = hand one out to everybody.

  Say: "Keep those sentences. Every math rule today is named after one of them, and the name tells you exactly what the rule lets you do."

0:03 to 0:13 Mini-lesson: one word, one property, one test

Commute. Numbers commute when they trade places: 17 + 48 and 48 + 17 both make 65; 6 × 9 and 9 × 6 both make 54. Say the name and the word: "commutative property, the numbers commute, they swap places." Now the test: does 10 − 4 equal 4 − 10? (6 vs. −6. No.) Does 12 ÷ 4 equal 4 ÷ 12? (3 vs. $\frac{1}{3}$. No.) Subtraction and division do not commute.

Associate. Parentheses say who is partnered with whom. (18 + 7) + 3 and 18 + (7 + 3) both make 28; the second grouping is easier because 7 + 3 is 10. (2 × 9) × 5 and 2 × (9 × 5) both make 90. Say: "associative property, you can change who the numbers associate with, who is grouped first." The test: (10 − 4) − 3 = 3 but 10 − (4 − 3) = 9. (24 ÷ 6) ÷ 2 = 2 but 24 ÷ (6 ÷ 2) = 8. Subtraction and division do not associate.

Distribute. 7 × (10 + 3): the 7 gets handed out to the 10 and to the 3: 7 × 10 + 7 × 3 = 70 + 21 = 91. Check the other way: 7 × 13 = 91. Say: "distributive property, the multiplier is distributed to every term inside." Then with a letter: 4(x + 5) = 4x + 20. Ask: "Did the 4 get handed to everybody?" (Yes: to x and to 5.) Common error: 4x + 5, where the 5 was skipped. "Everyone gets one."

Point out the direction: multiplication distributes over addition, not the reverse. 7 + (10 × 3) is not (7 + 10) × (7 + 3).

0:13 to 0:17 The big reveal: the two operations of algebra

Draw a scoreboard with four columns (add, subtract, multiply, divide) and two rows (commutes? associates?). Fill it with the student: addition yes yes, multiplication yes yes, subtraction no no, division no no.

Say: "This is why algebra runs on addition and multiplication. They are the operations that let us move things around. So what do we do with subtraction and division?" Bring back Lesson 1: subtracting 4 is adding the opposite, −4. Dividing by 4 is multiplying by the reciprocal, $\frac{1}{4}$.

Show the disguise working: 10 − 4 = 10 + (−4), and now it commutes: (−4) + 10 = 6. Same answer both ways. 12 ÷ 4 = 12 × $\frac{1}{4}$, and $\frac{1}{4}$ × 12 = 3. Same answer both ways. "Once subtraction and division are written as adding and multiplying, every property works."

0:17 to 0:23 Practice

Open the Deep Dive 5 interactive page for this lesson (or Worksheet Parts A to D). Same buttons as Lesson 1: Hint, Show next step, Reveal answer, Hide help / start over.

While they work, ask for the pair every time: the property name and the ordinary word. "Which property? What does that word mean? So what are the numbers allowed to do?"

0:23 to 0:25 Exit ticket

Worksheet Part E, or an index card: (1) Why is it called the commutative property? (2) Show with numbers that subtraction is not commutative. (3) Rewrite 15 − 6 so that it commutes, and show it works.

Success looks like: "because the numbers commute / trade places"; any pair such as 8 − 3 = 5 but 3 − 8 = −5; 15 + (−6) = 9 and (−6) + 15 = 9.

Misconceptions to listen for

- **Memorizing the names as labels.** If the student can say "commutative" but not "the numbers swap places," go back to the warm-up sentence about commuting to work. The word is the definition.
- **Mixing up commutative and associative.** Commutative moves the numbers (order changes). Associative moves the parentheses (grouping changes) while the order stays put. Ask: "Did anything change places, or did only the partners change?"
- **Skipping a term when distributing.** $4(x + 5) = 4x + 5$ is the classic miss. "Distribute means everyone gets one. Did the 5 get one?"
- **Distributing where nothing needs distributing.** $3 \times (4 \times 5)$ is a chain of multiplication; there is nothing to hand out. Distributing only applies when the parentheses hold addition or subtraction.
- **"Subtraction commutes if I just remember to switch the sign."** That is the disguise doing the work. $10 - 4$ does not equal $4 - 10$; $10 + (-4)$ equals $(-4) + 10$. The property belongs to addition.

Support and stretch

- **Support:** Act it out. Two students holding number cards physically commute (swap seats). Three students in a row put an arm around a neighbor to show which pair associates. One student hands a card to everyone inside a taped-off box to distribute.
- **Stretch:** Does division distribute? $(20 + 8) \div 4 = 20 \div 4 + 8 \div 4$ works, because dividing by 4 is multiplying by $\frac{1}{4}$ and multiplication distributes. But $20 \div (4 + 1)$ is not $20 \div 4 + 20 \div 1$. Ask the student to explain why one direction works and the other does not.
- **Preview:** Every step of solving an equation in the next lesson will be justified by naming a property: "distributive, then additive inverse, then additive identity." This lesson gives the student the words.

Where this goes next

Lesson 3: solving one-step and two-step equations and naming the property that justifies each line. Lesson 4: simplifying expressions by combining like terms, which is the distributive property run backwards (factoring out the letter).