Commute, Associate, Distribute

The properties, and why they are named that. Commute means trade places, associate means change partners, distribute means hand one out to everyone. Only addition and multiplication pass every test.

1 THE VOCABULARY
Words we use

Commutative property	to commute is to travel back and forth: the two numbers trade places. $a + b = b + a$ and $a \times b = b \times a$.
Associative property	an associate is a partner: the numbers change partners, the parentheses move. $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$.
Distributive property	to distribute is to hand one out to everyone: the multiplier goes to every term inside. $a \times (b + c) = a \times b + a \times c$.
Identity (Lesson 1)	the do-nothing numbers: $a + 0 = a$ and $a \times 1 = a$.
Inverse (Lesson 1)	the undo partners: $a + (-a) = 0$ and $a \times 1a = 1$.
Passes the test	an operation passes when the property is true for every pair of numbers, not just a lucky one.

- Watch out for these.
- Subtraction does not commute: $5 - 3$ is not $3 - 5$. Division does not either.
- Subtraction does not associate: $(8 - 4) - 2 = 2$, but $8 - (4 - 2) = 6$.
- Distributing to only the first term: $3(x + 2)$ is $3x + 6$, not $3x + 2$.
- Mixing up commute (trade places) with associate (move the parentheses). Watch what moved: the numbers, or the grouping?

2 THE METHOD
The three tests

1. **COMMUTE: trade places** swap the two numbers. Still equal? $4 + 7 = 7 + 4$ yes. $4 - 7 = 7 - 4$ no.
2. **ASSOCIATE: change partners** move the parentheses. Still equal? $(2 + 3) + 4 = 2 + (3 + 4)$ yes. $(8 \div 4) \div 2 = 8 \div (4 \div 2)$ no.
3. **DISTRIBUTE: hand one out to everyone** the multiplier outside the parentheses goes to every term inside. $3(x + 2) = 3x + 6$.
4. **PUT THE OTHER TWO IN DISGUISE** subtracting is adding the opposite, dividing is multiplying by the reciprocal (Lesson 1). Then every property works again.

> **WHY ONLY ADDITION AND MULTIPLICATION**
> They are the two operations that pass every test. That is why algebra treats them as its only two moves and rewrites subtraction and division in disguise: $5 - 3$ becomes $5 + (-3)$, and $8 \div 4$ becomes 8×14.

3 NAME THAT PROPERTY
Every line named

$4 + 9 = 9 + 4$	commutative property of addition: the numbers traded places
$(2 \times 5) \times 3 = 2 \times (5 \times 3)$	associative property of multiplication: the parentheses moved, the factors changed partners
$6(x + 4) = 6x + 24$	distributive property: the 6 was handed to the x and to the 4
$x + 0 = x$	additive identity (Lesson 1): the do-nothing number 0
$7 + (-7) = 0$	additive inverse (Lesson 1): the opposite undoes
$5 - 2 = 5 + (-2)$	not a property: it is the disguise. Subtracting is adding the opposite, so the properties can be used

> **CHECK WITH NUMBERS**
> Not sure whether an operation passes? Try it with real numbers. $10 - 4 = 6$ but $4 - 10 = -6$, so subtraction fails the commute test. One failure is enough to fail.

> **SAY IT OUT LOUD**
> Commute: trade places. Associate: change partners. Distribute: hand one out to everyone. Only adding and multiplying pass.

Lesson 2 in one line: commute means trade places, associate means change partners, distribute means hand one out to everyone, and only addition and multiplication pass every test.