Proving Shapes with Coordinates

Students use slope, distance, and midpoint together to prove what shape a set of points makes, find a missing vertex from the shared midpoint of a rectangle's diagonals, and write proofs as clear sentences that name the tool and the result.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 11, Lesson 3. Closes Unit 11.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson (shape prover), pencil, graph paper, calculator
PRIOR KNOWLEDGE	Distance (Lesson 1), midpoint (Lesson 2), slope and the parallel and perpendicular tests (Unit 4). Names of quadrilaterals and triangles (elementary and middle school).
STANDARDS	NC.M1.G-GPE.4, NC.M1.G-GPE.5, NC.M1.G-GPE.6.
KEEP OUT	Two-column proofs. Trapezoids and kites. Simplifying radicals.

Learning goals

1. Match each shape property to the tool that proves it.
2. Prove a quadrilateral is a square, rectangle, rhombus, or parallelogram, and classify a triangle.
3. Find a missing vertex of a rectangle from the shared midpoint of its diagonals.
4. Write a proof as a few clear sentences that name the tool and the result.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Parallelogram, rectangle, rhombus, square	the checklist	Sides, angles, diagonals.
Diagonal	corner to opposite corner	Compare midpoints.
Prove	show it must be true	Sentences a reader can check.

> THE ONE SENTENCE TO SAY OUT LOUD
>
> Slope for angles, distance for sides, midpoint for diagonals. Name the tool, state the result.

Lesson timeline (45 minutes)

0:00 to 0:05	**Warm-up: three questions.** Hold up a segment on the board: how steep, how long, where is the middle? Name the tool for each.
0:05 to 0:18	**The checklist and MNPQ.** Read the checklist. Use the shape prover on MNPQ: distance on all four sides, slope on two neighbors. Decide: a square. Say the evidence aloud.
0:18 to 0:30	**The missing corner and the proof sentence.** Midpoint of AC, step from B to D. Check angle B with slopes. Write the model sentence together. Then the triangle in the prover.
0:30 to 0:41	**Practice.** The practice problems or Worksheet Parts B to E. Same buttons: Hint, Show next step, Reveal answer, Hide help. Insist on sentences that name the tool.
0:41 to 0:45	**Exit ticket.** Worksheet Part F. Success looks like the right tool for each property and a square with both reasons.

Misconceptions to listen for

- **It looks like a square, so it is one.** Measure and check an angle.
- **One right angle makes a rectangle.** It must also be a parallelogram.
- **Four equal sides make a square.** That is a rhombus; check an angle.
- **A proof needs a table.** Clear sentences with the tool named are enough.
- **Perpendicular slopes are opposites.** They are negative reciprocals: flip and change the sign.

Support and stretch

Support

Give a three-column organizer (sides, angles, diagonals) and fill it with the prover's table. Provide the model sentence with blanks: ___ and ___ have slopes ___ and ___, which multiply to –1, so angle ___ is a right angle.

Stretch

Ask the student to make up four points that form a rhombus that is not a square and prove it. Ask whether a quadrilateral with equal diagonals must be a rectangle (no: an isosceles trapezoid has them too).

Where this goes next

Unit 12: descriptive statistics. The mean comes back, the same average used for the midpoint, and the square-then-square-root move from the distance formula returns when we measure how spread out data is.

> THE HABIT TO BUILD
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> Sides by distance, angles by slope, diagonals by midpoint, then write the reason in a sentence.