Recursive and Explicit, Both Ways

Students move from Now-Next wording to formal recursive notation, read both notation styles, translate recursive to explicit and explicit to recursive for arithmetic and geometric sequences, and explain when each form is handier.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 9, Lesson 3.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson, pencil
PRIOR KNOWLEDGE	Arithmetic and geometric sequences and both kinds of rule (Lessons 2 and 3). Function notation (Unit 2).
STANDARDS	NC.M1.F-BF.2, NC.M1.F-IF.3.
KEEP OUT	An explicit formula for the Fibonacci numbers. Rules that both add and multiply.

Learning goals

1. Write a Now-Next rule in formal notation with the initial term.
2. Read function notation and sequence notation for the term, the previous term, and the next term.
3. Translate recursive to explicit and explicit to recursive, arithmetic and geometric.
4. Explain when each form is handier, using the 40th-term race and the Fibonacci numbers.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Recursive rule	the stairs	Start plus next-from-now.
Explicit rule	the elevator	Any term from n.
Initial term	a_1	The start.
a_{n+1}, a_{n-1}	next, previous	Say: a sub n plus 1, a sub n minus 1.

> THE ONE SENTENCE TO SAY OUT LOUD
>
> Stairs for recursive, elevator for explicit. Find the start, find d or r, write the other form.

Lesson timeline (45 minutes)

0:00 to 0:05	Warm-up: Now-Next. Next = Now + 12, starting at 4. List five terms. Ask: what is the 40th? Let the student feel how long the stairs are.
0:05 to 0:15	Formal notation. Now is a sub n, Next is a sub n plus 1, start is a sub 1. Read the notation table aloud, both columns. Show the now-from-previous form too.
0:15 to 0:28	Translate both ways. Three steps each way, one arithmetic and one geometric example for each direction. Simplify 4 + 12(n – 1) to 12n – 8 and note the y-intercept.
0:28 to 0:34	When each is handier. The race to term 40: 39 steps against one calculation. Then Fibonacci: simple recursive rule, no easy explicit one.
0:34 to 0:41	Practice. The converter practice or Worksheet Parts C and D. Same buttons: Hint, Show next step, Reveal answer, Hide help.
0:41 to 0:45	Exit ticket. Worksheet Part F. Success looks like 5n – 3 and $a_1 = 4$ with times 3.

Misconceptions to listen for

- **A recursive rule without a start.** It needs a_1.
- **In 3n + 2 the start is 2.** Put in n = 1: the start is 5.
- **a sub n plus 1 means a sub n, plus 1.** The whole n + 1 is the subscript: the next term.
- **Explicit is always better.** Recursive is easier to read, and Fibonacci needs it.
- **r to the n.** n − 1 multiplications.

Support and stretch

Support

Give a three-box template for each direction. Read every subscript aloud. Let the student list the first four terms from both rules to confirm they match.

Stretch

Ask for an explicit rule for 1, 4, 9, 16 (n squared) and whether it has a simple recursive rule (add the next odd number). Ask for the 12th Fibonacci number by stairs.

Where this goes next

Lesson 5: building sequences from the real world. Decide arithmetic or geometric from a story or a table, write the rule, and answer the question.

> THE HABIT TO BUILD
>
> Find the start, find d or r, write the other form, and check by listing the first few terms.