Recursive and Explicit, Both Ways

Recursive: the start and the next-from-now rule. Explicit: any term from n.
To translate: find the start, find d or r, write the other form.

Name _____ Date _____

Words we use say the math word, then the plain English

WORD	PLAIN ENGLISH	WHAT YOU DO
Recursive rule	How to get the next term from the one before. The stairs. In plain words: Next = Now + 12.	It needs the starting term.
Explicit rule	How to find any term straight from its term number. The elevator.	Put in n and calculate once.
Initial term	a_1, the starting point a recursive rule needs.	Without it you cannot start climbing.
a_{n+1}, said a sub n plus 1	The next term, the one after a_n. Same as f(n + 1).	In Now-Next words: Next.
a_{n-1}, said a sub n minus 1	The previous term, the one before a_n. Same as f(n – 1).	Each term is built from it.
Translate	Change a rule from one form to the other without changing the list.	Find the start; find d or r; write the other form.

Part A Now-Next to formal notation

NOW-NEXT	FORMAL RECURSIVE RULE	FIRST FOUR TERMS
Next = Now + 7, starting at 3		
Next = Now × 2, starting at 5		
Next = Now – 4, starting at 30		

Part B The notation table

WHAT IT IS	FUNCTION NOTATION	SEQUENCE NOTATION	SAY IT
the term	f(n)		
the previous term		a_{n-1}	
the next term			a sub n plus 1

Part C Recursive to explicit

RECURSIVE RULE	START	ADDS D OR MULTIPLIES R	EXPLICIT RULE
$a_{n+1} = a_n + 12,\ a_1 = 4$			
$a_{n+1} = 3 \cdot a_n,\ a_1 = 5$			
$a_1 = 10,\ a_n = a_{n-1} - 3$			

1. Simplify the first explicit rule. What is the y-intercept? Why is it not 4?

Part D Explicit to recursive

EXPLICIT RULE	PUT IN N = 1	D OR R	RECURSIVE RULE
$a_n = 3n + 2$			
$a_n = 6 \cdot 2^{n-1}$			
$a_n = 20 - 4(n - 1)$			

Part E Stairs or elevator?

1. Find the 40th term of $a_{n+1} = a_n + 12$, $a_1 = 4$. How many steps would the recursive rule take? How many calculations does the explicit rule take?

2. The Fibonacci numbers are 0, 1, 1, 2, 3, 5, 8, ... Write the next three. Describe the recursive rule in words.

Part F Exit ticket

1. Next = Now + 5, starting at 2: formal rule, explicit rule, simplified.

2. $a_n = 4 \cdot 3^{n-1}$: the recursive rule.

Answer key

Part A. $a_{n+1} = a_n + 7$, $a_1 = 3$; 3, 10, 17, 24. $a_{n+1} = 2 \cdot a_n$, $a_1 = 5$; 5, 10, 20, 40. $a_{n+1} = a_n - 4$, $a_1 = 30$; 30, 26, 22, 18.

Part B. a_n, a sub n. f(n – 1), a sub n minus 1. f(n + 1), a_{n+1}.

Part C. 4, adds 12, 4 + 12(n – 1). 5, multiplies by 3, $5 \cdot 3^{n-1}$. 10, adds –3, 10 – 3(n – 1). 1. 12n – 8; y-intercept –8, the zeroth term (4 – 12); the start 4 is at n = 1.

Part D. 5, d = 3, $a_1 = 5$ and $a_{n+1} = a_n + 3$. 6, r = 2, $a_1 = 6$ and $a_{n+1} = 2 \cdot a_n$. 20, d = –4, $a_1 = 20$ and $a_{n+1} = a_n - 4$.

Part E. 1. 472; 39 steps; one calculation: $4 + 12 \times 39$. 2. 13, 21, 34; each term is the sum of the two before it.

Part F. 1. $a_{n+1} = a_n + 5$, $a_1 = 2$; $a_n = 2 + 5(n - 1) = 5n - 3$. 2. $a_1 = 4$, $a_{n+1} = 3 \cdot a_n$.