Residuals, Checking the Fit

Students compute residuals as actual minus predicted, read their signs, make and read residual plots, and use the scatter plot, r, and the residual plot together to decide whether a line is an appropriate model.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 10, Lesson 2.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson (regression tool with residual view), pencil, graph paper. A graphing calculator can be used instead of the site's tool.
PRIOR KNOWLEDGE	The line of best fit and predicted values (Lesson 2). r (Lesson 1). Subtracting signed numbers.
STANDARDS	NC.M1.S-ID.6b, NC.M1.S-ID.8.
KEEP OUT	Exponential regression (Lesson 4). Formal tests of fit.

Learning goals

1. Compute a residual as actual minus predicted and say whether the point is above or below the line.
2. Make a residual plot from a table.
3. Decide from a residual plot whether a line is appropriate: random scatter or a pattern.
4. Explain why a big r does not prove a line is right, and use the scatter plot, r, and the residual plot together.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Residual	actual – predicted	The model's error for one point.
Residual plot	residuals against x	A flat line at 0.
Random scatter	no pattern	A line fits.
Pattern	a U or a hill	Try another family.

> THE ONE SENTENCE TO SAY OUT LOUD
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> Every model is a little bit wrong. Actual minus predicted. Random scatter, keep the line; a pattern, try another family.

Lesson timeline (45 minutes)

0:00 to 0:05	Warm-up: how far off? A model predicts 7,744 and the real value is 7,874. How far off, and which way? Name the residual.
0:05 to 0:16	Mini-lesson: residuals. Actual minus predicted. Positive is above. The tuition table: fill in predicted and residual together. Show the vertical segments in the tool.
0:16 to 0:26	The residual plot. The rule: random scatter or a pattern. Tuition: random. Sno-cones: r of about 0.89 and random scatter.
0:26 to 0:33	The surprise. Gnats with a line: r of about 0.98, residuals plus, minus, minus, plus, plus. A curve. Use all three together.
0:33 to 0:41	Practice. The practice problems or Worksheet Parts A to D. Same buttons: Hint, Show next step, Reveal answer, Hide help.
0:41 to 0:45	Exit ticket. Worksheet Part E. Success looks like –6, below the line, and naming all three checks.

Misconceptions to listen for

- **Predicted minus actual.** Actual minus predicted, so positive means above.
- **Small residuals mean a good fit.** Look for a pattern too.
- **A big r proves a line.** The gnats have r of 0.98 and curve.
- **A residual is a distance, so it is never negative.** It has a sign: below the line is negative.
- **The residual plot should look like the data.** It should look like nothing: random.

Support and stretch

Support

Use two colors: one for points above the line, one for below. Have the student say too low or too high for each residual. Build the residual plot by hand once before using the tool.

Stretch

Ask why the residuals of a least squares line always add to zero (try it on the tuition data with the tool's own line). Ask what a hill-shaped residual plot says about the data.

Where this goes next

Lesson 4: exponential regression. When the data multiplies, fit the exponential family instead, and read a and b in the story.

> THE HABIT TO BUILD
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> Actual minus predicted, read the sign, plot the residuals, and look for a pattern before trusting the line.