Using Slopes to Check Shapes

Students use the slopes of a shape's sides to prove parallel sides and right angles from the corner points, classifying rectangles, parallelograms, and right triangles, and learn what slope cannot prove until distance and midpoint arrive in Unit 11.

TIME	40 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 4, Lesson 3.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson, pencil, graph paper, ruler, a paper corner
PRIOR KNOWLEDGE	Slope from two points (second minus first). Parallel means equal slopes; perpendicular means product –1 (Lessons 1 to 3).
STANDARDS	NC Math 1 NC.M1.G-GPE.4 (use coordinates to prove simple geometric theorems algebraically, e.g. a figure is a rectangle) and NC.M1.G-GPE.5. Distance and midpoint (G-GPE.6, 7) are deferred to Unit 11.
KEEP OUT	Distance, midpoint, and anything about side lengths. Say plainly that those come in Unit 11.

Learning goals

1. Compute the slope of every side of a shape from its corner points, taken in order.
2. Prove a triangle is a right triangle by finding two sides whose slopes multiply to -1, and name the right angle.
3. Prove four points make a rectangle (opposite sides parallel, neighbors perpendicular) or only a parallelogram, or neither.
4. Say what slope can prove and what it cannot (lengths), and where the missing tools come (Unit 11).

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Right triangle	one right angle	One pair of sides with product –1.
Rectangle	four right angles, opposite sides parallel	Equal slopes across; product –1 at the corners.
Parallelogram	opposite sides parallel	Equal slopes across only.
Prove with algebra	computed numbers, not a look	Every claim rests on a slope you found.
Distance, midpoint	Unit 11	For lengths and centers.

> THE ONE SENTENCE TO SAY OUT LOUD
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> Corners in order, every side slope. Equal slopes mean parallel. Product negative one means a right angle. That proves it. Lengths wait for Unit 11.

Lesson timeline (40 minutes)

0:00 to 0:05	Warm-up: does it look like a rectangle? Plot A(–3, 1), B(1, 3), C(3, –1), D(–1, –3). Ask whether it is a rectangle and how the student knows. Looking is a guess; today we prove it.
0:05 to 0:15	Mini-lesson: the two tests. Compute the four slopes together, in order around the shape. Opposite sides equal: parallel. Neighbors multiply to –1: right angles. Say the word proof. Then the right triangle P, Q, R: three slopes, one product of –1, right angle at Q.
0:15 to 0:25	Plot it, then test it. The two interactive grids: plot the corners, then type the slopes and decide. The second one is deliberately not a right triangle, though it looks close.
0:25 to 0:36	Practice. The practice problems or Worksheet Parts A and B. Same buttons: Hint, Show next step, Reveal answer, Hide help. Insist on all the slopes being written before any decision.
0:36 to 0:40	What slope cannot do, and the exit ticket. Ask whether the rectangle is a square. Slope cannot say; Unit 11 brings distance. Exit ticket: Worksheet Part D. Success looks like: right angle at B; slope says nothing about length.

Misconceptions to listen for

- **It looks like a rectangle, so it is.** Compute. The second interactive triangle looks right but is not.
- **Sides out of order.** AC is a diagonal, not a side. Go around.
- **One sign slip.** Second minus first, top and bottom, every time.
- **Parallel sides means rectangle.** That is a parallelogram. Rectangles also need the product –1.
- **Slopes can show a square.** They cannot; equal sides need distance.
- **Checking one corner and stopping.** Check the corners you need for the claim (all four for a rectangle, though parallel opposite sides plus one right angle forces the rest).

Support and stretch

Support

Plot every shape on graph paper before computing. Label corners in order with a pencil arrow around the shape. Use a table with four rows: side, rise, run, slope.

Stretch

Ask for a fourth point that makes three given points into a rectangle. Ask whether a shape with two pairs of equal slopes but no right angles can still have a right angle somewhere else (no). Ask what would be needed to prove a rhombus (equal sides: distance, Unit 11).

Where this goes next

Unit 5: graphing linear inequalities in two variables. Still on the coordinate plane: graph a line, then shade the side that makes the inequality true. Unit 11 later adds distance and midpoint to finish shape proofs.

> THE HABIT TO BUILD
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> Corners in order, every side slope written down, then the two tests. Say what you proved and what you did not.