Using Slopes to Check Shapes

From the corners of a shape, the side slopes prove which sides are parallel (equal slopes) and which meet at a right angle (slopes multiply to -1). Enough for a rectangle or a right triangle. Lengths wait for Unit 11.

1 THE VOCABULARY
Words we use

Right triangle one right angle: two sides with slopes that multiply to –1.

Rectangle four right angles, opposite sides parallel: equal slopes across, product –1 at each corner.

Parallelogram opposite sides parallel: equal slopes across, no right-angle requirement.

Prove with algebra back every claim with a computed slope, not with how it looks.

Distance, midpoint lengths and centers: Unit 11. Not slope's job.

- Watch out for these.
- Going around the shape out of order. Label the corners in order and take the sides in order.
- A sign slip in one slope. Second minus first, on top and on the bottom.
- Calling it a rectangle because it looks like one. Compute all four slopes.
- Testing only one corner of a rectangle. Neighbors at every corner should multiply to –1 (though once opposite sides are parallel, one right angle forces the rest).
- Claiming a square from slopes alone. Slope cannot compare lengths.

2 THE METHOD
Testing a shape with slope

1. **LABEL THE CORNERS IN ORDER** A, B, C, D around the shape. Plot them if you can.
2. **COMPUTE EVERY SIDE SLOPE** rise over run, second minus first both times: AB, BC, CD, DA.
3. **PARALLEL TEST** opposite sides equal slopes? (Parallelogram, and half of rectangle.)
4. **RIGHT-ANGLE TEST** neighboring sides multiply to –1? (Right triangle: any one pair. Rectangle: at the corners.)
5. **SAY WHAT YOU PROVED, AND WHAT YOU DID NOT** parallel and perpendicular, yes. Equal lengths, not yet: that is distance, Unit 11.

WHY THIS IS A PROOF

A picture can fool the eye; a slope cannot. Each test is a computed number compared with another computed number. If the numbers pass, the claim is true for these exact points.

3 A RIGHT TRIANGLE
P(–2, –1), Q(4, 2), R(2, 6)

PQ: (2 – (–1))/(4 – (–2)) = 3/6 = 1/2	rise over run
QR: (6 – 2)/(2 – 4) = 4/(–2) = –2	rise over run
RP: (–1 – 6)/(–2 – 2) = –7/(–4) = 7/4	rise over run
1/2 × (–2) = –1	PQ and QR are perpendicular
right angle at Q	the corner where PQ and QR meet. PQR is a right triangle

ON THE GRID

Plot the three points and hold a paper corner against Q. It fits. The other two corners do not.

SAY IT OUT LOUD

Corners in order. Every side slope. Equal means parallel, product negative one means a right angle. Lengths wait for Unit 11.