Using Slopes to Check Shapes

From the corners of a shape, compute the slopes of its sides. Equal slopes mean parallel sides. Slopes that multiply to -1 mean a right angle. That proves rectangles and right triangles with algebra alone.

Name _____ Date _____

Words we use

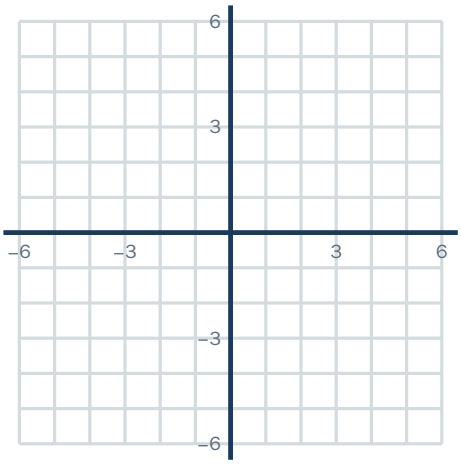
say the math word, then the plain English

WORD	PLAIN ENGLISH	WHAT YOU DO
Right triangle	A triangle with one right angle.	Find two sides whose slopes multiply to –1; the right angle is where they meet.
Rectangle	Four right angles, and opposite sides parallel.	Opposite sides: equal slopes. Neighbors: slopes multiply to –1.
Parallelogram	Opposite sides parallel, corners not necessarily square.	Equal slopes for opposite sides only.
Parallel, perpendicular	Same slope; slopes that multiply to –1.	The two tests from Lessons 1 and 2, now applied to sides.
Prove with algebra	Back up a claim about a shape with numbers you computed, not with how it looks.	Compute every slope you need and show the test passes.
Distance, midpoint	The length of a side, and its center point. Unit 11.	Needed to show sides are equal (a square) or to find centers. Not slope's job.

Part A Right triangle?

Plot the points, compute the three side slopes, and decide. Name the right angle if there is one.

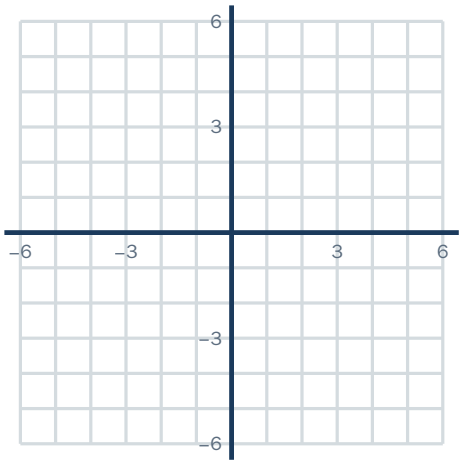
1. A(0, 0), B(4, 2), C(2, 6)



AB ____ BC ____ CA ____

Right triangle? ____ Right angle at ____

2. D(–3, 1), E(1, 3), F(3, 0)



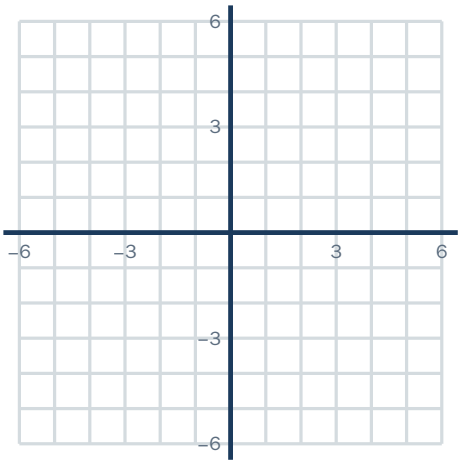
DE ____ EF ____ FD ____

Right triangle? ____ Right angle at ____

Part B Rectangle, parallelogram, or neither?

Compute the four side slopes in order. Test opposite sides (equal?) and neighbors (product –1?).

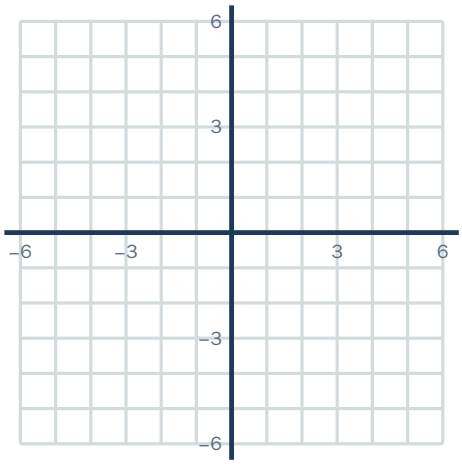
1. A(–3, 1), B(1, 3), C(3, –1), D(–1, –3)



AB ____ BC ____ CD ____ DA ____

Answer: _____

2. P(0, 0), Q(4, 1), R(6, 4), S(2, 3)



PQ ____ QR ____ RS ____ SP ____

Answer: _____

1. 3. W(–4, 0), X(0, 2), Y(2, –2), Z(–2, –4). Slopes and answer:

Part C In your own words

1. What does it mean to prove a shape is a rectangle with algebra, instead of by looking?

2. What can slope prove about a shape, and what can it not prove yet? Which unit brings the missing tool?

3. Why must you take the sides in order around the shape?

Part D Exit ticket

1. A(0, 0), B(2, 4), C(6, 2): right triangle? Where is the right angle?

2. Why can slope not tell you whether a rectangle is a square?

Answer key

Part A. 1. AB 1/2, BC –2, CA 3. Yes, right angle at B. 2. DE 1/2, EF –3/2, FD –1/6. Products: –3/4, 1/4, –1/12. No.

Part B. 1. 1/2, –2, 1/2, –2: rectangle. 2. 1/4, 3/2, 1/4, 3/2: parallelogram only (1/4 × 3/2 = 3/8). 3. 1/2, –2, 1/2, –2: rectangle.

Part C. 1. Compute the side slopes and show opposite sides equal and neighbors multiply to –1. 2. Parallel sides and right angles; not lengths (equal sides, squares) or centers; Unit 11 brings distance and midpoint. 3. Otherwise you compare the wrong sides as opposite or neighbors.

Part D. 1. AB 2, BC –1/2, CA 1/3. 2 × (–1/2) = –1: yes, at B. 2. Slope says nothing about length; that needs distance.