Solving by Elimination

Students learn elimination: add the equations so one variable cancels, with the reasoning (adding equal things keeps equality; opposite coefficients make a zero appear), multiplying one or both equations first, and a guide to choosing the easiest method.

TIME	45 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Unit 6, Lesson 2.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive page for this lesson, pencil
PRIOR KNOWLEDGE	Additive inverses and the additive identity (Unit 1, Lesson 1). Multiplying both sides of an equation (Unit 1). Substitution (Lesson 2).
STANDARDS	NC Math 1 NC.M1.A-REI.5 (replacing one equation by the sum of it and a multiple of the other produces a system with the same solutions) and A-REI.6.
KEEP OUT	Number of solutions (Lesson 4), beyond noticing a strange final line.

Learning goals

1. Explain why adding two equations is allowed and why opposite coefficients make a variable disappear.
2. Solve by elimination when the coefficients are already opposites or equal, when one equation must be multiplied, and when both must.
3. Back-substitute into either original and check the pair in both.
4. Choose among graphing, substitution, and elimination and say why.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Elimination	add to cancel	Linear combination.
Opposite coefficients	3 and –3	Additive inverses; they add to 0.
Multiply an equation	both sides	Unit 1 move.
Choose the method	see, alone, line up	Graph, substitute, eliminate.

> THE ONE SENTENCE TO SAY OUT LOUD
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> Line them up. Make one variable's coefficients opposites. Add, and the zero appears. Solve, back-substitute, check both.

Lesson timeline (45 minutes)

0:00 to 0:05	Warm-up: what adds to zero? $3y + (-3y)$? Unit 1, Lesson 1. Then show $2x + 3y = 12$ and $4x - 3y = 6$ and ask what happens if we add them straight down.
0:05 to 0:17	Mini-lesson: why, then the steps. Say why adding is allowed (equal plus equal). Solve the warm-up system through to the check. Then $x + 2y = 7$ and $3x - y = 7$: no opposites yet, multiply the second by 2 (both sides, every term). Then a both-multiply case.
0:17 to 0:24	Choosing the method. Three systems on the board: one with y alone, one with opposites ready, one to see. Ask which road and why. Say: every road turns two variables into one.
0:24 to 0:40	Practice. The practice problems or Worksheet Parts A and B. Same buttons: Hint, Show next step, Reveal answer, Hide help. Require the multiplied equations to be written out in full.
0:40 to 0:45	Exit ticket. Worksheet Part D. Success looks like (5, 3), and multiply the second by 5.

Misconceptions to listen for

- **Multiplying only the left side.** Both sides, every term, or the equation is no longer true.
- **Adding equal coefficients.** 2y + 2y is 4y, not 0. Subtract, or multiply one by –1.
- **Sign slips when multiplying by a negative.** Every term changes sign.
- **Stopping at one variable.** Back-substitute.
- **Thinking one method is always right.** Choose by the shape of the system.

Support and stretch

Support

Write the two equations in a table with columns for x, y, and the number so lining up is automatic. Circle the two terms that will cancel before adding. Keep multipliers small.

Stretch

Ask why multiplying by –2 rather than 2 was needed in the both-multiply case. Give a system in y = mx + b form and ask the student to rewrite it for elimination. Ask what elimination gives when the lines are parallel (Lesson 4).

Where this goes next

Lesson 4: how many solutions? One (lines cross), none (parallel: the algebra ends in a false line), or infinitely many (the same line: a true line). And predicting it from m and b before solving.

> THE HABIT TO BUILD
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> Before adding, point to the two terms that will cancel and say they are opposites.