



Solving Equations: Undo to Get the Letter Alone

A 25-minute lesson for students about 12 years old. It teaches solving one-step and two-step equations by translating first (Step 1), then undoing what is added to the letter and its coefficient, naming the undo partner (inverse) and the do-nothing number (identity) at every step. It puts Lessons 1, 2, and 3 to work.

TIME	25 minutes (one student with a parent or tutor, or a small group)
WHO	Math 1 (algebra readiness), about age 12. Follows Lesson 1 (identity and inverse), Lesson 2 (the properties), and Lesson 3 (Step One: translate).
MATERIALS	Student worksheet (PDF), the Deep Dive 5 interactive practice page for this lesson, pencil, whiteboard or scrap paper
PRIOR KNOWLEDGE	Lesson 1: the do-nothing numbers (0 and 1) and the undo partners (opposite and reciprocal). Lesson 2: the properties, and why only addition and multiplication pass them. Lesson 3: subtracting is adding the opposite, dividing is multiplying by the reciprocal, and the invisible 1s.
STANDARDS	Builds toward NC Math 1 NC.M1.A-REI.1 (justify each step of solving an equation) and NC.M1.A-REI.3 (solve linear equations in one variable). Rests on grade 6 and 7 work solving equations of the form $px + q = r$ (6.EE.7, 7.EE.4).

Learning goals

1. Start every equation with Step 1: translate subtraction and division, and write in the invisible 1s, so that every term shows its real coefficient.
2. Solve a one-step equation and name, out loud, the undo partner (inverse) used and the do-nothing number (identity) that appears: add the opposite to make 0, or multiply by the reciprocal to make 1.
3. Solve a two-step equation by undoing in the reverse order it was built: undo what is added first, then the coefficient, doing the same thing to both sides so the balance holds.
4. Justify each line by the property that makes it true, and check a solution by putting it back into the original equation.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student can do it unprompted.

TERM	SYMBOLS	PLAIN ENGLISH
Translate (Step 1)	$a - b = a + (-b)$ $a \div b = a \times \frac{1}{b}$	Rewrite subtraction and division as addition and multiplication, and write in the invisible 1s. Do it first, every time.
Coefficient	$3x$ means $3 \cdot x$ x means $1x$	The number on the variable. It means the variable is multiplied by that number. We usually do not write the 1, and anything algebra leaves you to do in your head is a top cause of mistakes. So write it in.
Equation	left = right	A balance that says the two sides are equal. Keep them equal at every step.
Solve (isolate the variable)	$x = ?$	Find the value of the letter that makes the equation true. You do it by getting the letter by itself.
Inverse operation (the undo partner)	+ undoes adding a negative $\times$ by the reciprocal undoes $\times$	The move that undoes another. After Step 1 there are only two: add the opposite, or multiply by the reciprocal.
Additive inverse (the opposite)	$a + (-a) = 0$	Add it to undo what is added. It makes the additive identity 0 appear, so the letter is left alone.
Multiplicative inverse (the reciprocal)	$a \times \frac{1}{a} = 1$	Multiply by it to undo the coefficient. It makes the multiplicative identity 1 appear, so the letter is left alone.
Do the same to both sides	if $a = b$, then $a + c = b + c$	Whatever you add or multiply on one side, do to the other, so the balance never tips.

THE ONE SENTENCE TO SAY OUT LOUD

Solving is undoing. Step 1, translate: turn subtraction into adding the opposite, division into multiplying by the reciprocal, and write in the invisible 1s. Then undo what is added to the letter by adding the opposite (a 0 appears), and undo the coefficient by multiplying by the reciprocal (a 1 appears), doing the same thing to both sides so the balance never tips. Every line of solving is one of the properties from Lessons 1 to 3.

Lesson timeline (25 minutes)

- 0:00 to 0:03** **Warm-up: name the coefficient.** Write these four equations and, before solving anything, ask three questions about each: what is the coefficient of x , what is added to x , and what would undo each? (a) $x + 5 = 12$ (b) $x - 4 = 9$ (c) $3x = 21$ (d) $\frac{x}{2} = 6$. Expected: (a) coefficient 1 (invisible), +5 added, undo by adding -5 ; (b) coefficient 1, -4 added (after Step 1), undo by adding $+4$; (c) coefficient 3, nothing added, undo by multiplying by $\frac{1}{3}$; (d) coefficient $\frac{1}{2}$ (after Step 1), nothing added, undo by multiplying by 2.
- Say: "Every equation is the same job. Name the coefficient and what is added. Solving just means undoing them." If a student says the coefficient of x is "nothing," stop there: that is the invisible 1, and leaving it in your head is where mistakes start.
- 0:03 to 0:11** **Mini-lesson: Step 1, then one step, then two.**
- Step 1 always.** Before solving, translate. Write $2x - 6 = 8$ and rewrite it as $2x + (-6) = 8$. Ask: "What is the coefficient of x ?" (2). "Any invisible 1s here?" (No, but $x - 6 = 8$ would be $1x + (-6) = 8$.)
- One step.** Solve $6x = 42$ together, saying it out loud: "The coefficient is 6, so undo it with the reciprocal, $\frac{1}{6}$. Do it to both sides. $\frac{1}{6} \times 6 = 1$, the do-nothing number, so x is alone. $x = 7$." Do $x + 8 = 20$ the same way: "coefficient 1, +8 added; add the opposite of 8, which is -8 ; $8 + (-8) = 0$, so x is alone; $x = 12$."
- Two steps.** Solve $3x + 4 = 19$. Ask: "Two things here: a coefficient of 3, and +4 added. Which comes off first?" Use the getting-dressed picture: socks on, then shoes; to undo, the shoes come off first. The +4 is the outside layer, so undo it first (add -4 , which makes 0): $3x = 15$. Then undo the coefficient with $\frac{1}{3}$ (which makes 1): $x = 5$. Draw a balance the whole time: whatever you do to one side, do to the other, or the scale tips.
- 0:11 to 0:16** **The order that never changes.** State the rule: Step 1 translate, then undo what is added, then undo the coefficient, then check. Work two together: $2x - 5 = 11$ (Step 1: $2x + (-5) = 11$; add 5 to both sides; then multiply by $\frac{1}{2}$; $x = 8$) and $\frac{x}{4} + 2 = 6$ (Step 1: $\frac{1}{4}x + 2 = 6$; add -2 ; then multiply by 4; $x = 16$).
- Then a parentheses one, $3(x + 2) = 21$: "distribute first (Lesson 2), then it is just a two-step equation." Finish by checking: put x back into the original and show both sides match.
- 0:16 to 0:22** **Practice.** Open the Deep Dive 5 interactive page for this lesson (or Worksheet Parts A to D). Same buttons as before: Hint, Show next step, Reveal answer, Hide help / start over.
- While they work, ask for the three questions every time: "What is the coefficient, and what is added? What undoes each? Which do-nothing number appeared?"
- 0:22 to 0:25** **Exit ticket.** Worksheet Part E, or on an index card: (1) In $4x + 5 = 25$, what is the coefficient of x , what is added, and which do you undo first? (2) Solve $3x - 6 = 9$, showing Step 1, and check your answer. (3) One sentence: why do we do the same thing to both sides?
- Success looks like: coefficient 4, +5 added, the +5 first; $x = 5$ with a check that $3(5) - 6 = 9$; any sentence about keeping the balance equal.

Misconceptions to listen for

- **Skipping Step 1.** Trying to "subtract 6 from both sides" of $2x - 6 = 8$ works on easy problems and falls apart on $x - (-3) = 5$. Translate first, then every undo is adding an opposite or multiplying by a reciprocal.
- **Leaving the invisible 1 in your head.** "The coefficient of x is nothing," so $5x - x$ becomes 5, or $x + 2x$ becomes $2x$. Write the 1. Anything algebra leaves you to do in your head is a top cause of mistakes.
- **Undoing in the wrong order.** Multiplying by the reciprocal before undoing what is added gives the wrong answer. Peel the outside layer first: undo what is added before you undo the coefficient.
- **Only changing one side.** The equation is a balance. If you add or multiply on the left, you must do the same on the right, or the two sides stop being equal.
- **Sign slips.** With a subtraction or a negative coefficient, Step 1 fixes most of them: " $2x - 5$ " is " $2x + (-5)$," so you undo it by adding $+5$. A coefficient of -2 is undone by multiplying by $-\frac{1}{2}$, and the sign stays with the reciprocal.
- **Forgetting a term when distributing.** $3(x + 2)$ is $3x + 6$, not $3x + 2$. Everyone inside the parentheses gets one (Lesson 2).
- **"Move it over and flip the sign" with no reason.** Moving a term to the other side and changing its sign is really adding the opposite to both sides. Say the reason out loud instead of memorizing a trick.
- **Not checking.** Put the answer back into the original equation. If both sides come out equal, it is right; if not, find the step where the balance tipped.

Support and stretch

Support

Use a real balance, or cups and counters, so "do the same to both sides" is something the student can see. Give the order as a checklist: (1) Step 1, translate and write the invisible 1s; (2) undo what is added; (3) undo the coefficient; (4) check. Have the student write the coefficient over every letter before starting, including the 1s.

Stretch

Clear a fraction coefficient by multiplying the whole equation by the reciprocal in one move. Preview an equation with the letter on both sides: gather the letters on one side first, then solve. Ask what happens if the letter disappears (no solution, or true for every number).

Where this goes next

Lesson 5: simplifying expressions by combining like terms (the coefficients add: $2x + 3x = 5x$, and $x + 2x = 1x + 2x = 3x$) and factoring, which is the distributive property run backwards. Same tools, longer problems. After that, equations with the variable on both sides, and one-variable inequalities, which solve the same way with one extra rule about the direction of the sign.

THE HABIT TO BUILD

Name the coefficient, even when it is 1. Name what is added. Undo the added part first, then the coefficient. Say the undo partner and the do-nothing number out loud every time. Check.