Solving Equations: Undo to Get the Letter Alone

Solving is undoing. Step 1, translate: subtraction becomes adding the opposite, division becomes multiplying by the reciprocal, and the invisible 1s get written. Then undo what is added, then undo the coefficient, doing the same thing to both sides.

1 THE VOCABULARY
Words we use

Translate (Step 1) rewrite subtraction as adding the opposite and division as multiplying by the reciprocal, and write in the invisible 1s: x is 1x.

Equation a balance that says two sides are equal. Keep them equal at every step.

Solve find the value of the letter that makes it true, by getting the letter by itself.

Coefficient the number on the variable: 3x is 3 times x. A lonely x has coefficient 1.

Inverse operation (undo partner) add the additive inverse (the opposite) to make 0 appear; multiply by the multiplicative inverse (the reciprocal) to make 1 appear.

Do the same to both sides whatever you add or multiply on one side, do to the other, or the balance tips.

- Watch out for these.
- Skipping Step 1. Translate first, then every undo is adding an opposite or multiplying by a reciprocal.
- Leaving the invisible 1 in your head: x + 2x is 1x + 2x = 3x.
- Undoing in the wrong order: undo what is added before you undo the coefficient.
- Changing only one side. The equation is a balance.
- Not checking. Put the answer back in the original: both sides must match.

2 THE METHOD
The order that never changes

1. **STEP 1: TRANSLATE** 2x – 6 = 8 becomes 2x + (–6) = 8. Write the invisible 1s.
2. **UNDO WHAT IS ADDED** add the additive inverse (the opposite) to both sides: 2x + (–6) + 6 = 8 + 6, so 2x = 14. A 0 appeared.
3. **UNDO THE COEFFICIENT** multiply both sides by the multiplicative inverse (the reciprocal): 12 × 2x = 12 × 14, so x = 7. A 1 appeared.
4. **CHECK** put it back: 2(7) + (–6) = 8. Both sides match.

> **WHY THIS ORDER**
> An equation is built from the inside out: first the coefficient multiplies the letter, then something is added. To undo it you peel the outside layer first, like taking off shoes before socks: undo what is added, then undo the coefficient.

3 TWO STEPS, EVERY LINE NAMED
3x + 4 = 19

3x + 4 = 19	the original: coefficient 3, and 4 is added. Nothing to translate
3x + 4 + (–4) = 19 + (–4)	add –4, the additive inverse (the opposite) of 4, to both sides
3x + 0 = 15, so 3x = 15	4 + (–4) = 0, and 3x + 0 = 3x by the additive identity (the do-nothing number 0)
13 × 3x = 13 × 15	multiply both sides by 13, the multiplicative inverse (the reciprocal) of the coefficient 3
1x = 5, so x = 5	13 × 3 = 1, and 1x = x by the multiplicative identity (the do-nothing number 1)
check: 3(5) + 4 = 19	put 5 back into the original. 15 + 4 = 19. Both sides match

> **A PARENTHESES ONE**
> 3(x + 2) = 21: distribute first (Lesson 2) to get 3x + 6 = 21, and it is the same two-step job. Add –6 to both sides, then multiply by 13: x = 5. Check: 3(5 + 2) = 21.

> **SAY IT OUT LOUD**
> Translate. Undo what is added. Undo the coefficient. Same thing to both sides. Check.

Lesson 4 in one line: Step 1 translate, then undo what is added, then undo the coefficient, doing the same to both sides, and check by putting the answer back.