

Step One: Translate Subtraction and Division

Only addition and multiplication follow the properties. So before you solve, turn every subtraction into adding the opposite, and every division into multiplying by the inverse.

The big idea

The properties (commute, associate, distribute, identity, inverse) belong to **addition and multiplication**. Subtraction and division do not play fair ($5 - 2$ is not $2 - 5$). So treat them as nicknames and translate them away. Then every property is yours for the rest of the problem.

The one move, in two places

Change the operation to its partner, and replace the number with its inverse.

Nickname	Real form	Partner you swap in
$a - b$	$a + (-b)$	the opposite of b
a / b	$a \times (1/b)$	the reciprocal of b

Say it out loud: minus becomes plus with the opposite; divide becomes times with the reciprocal.

Multiplying fractions

Because 'divide by 2' becomes 'multiply by $1/2$ ', you will multiply fractions a lot. Multiply the tops, multiply the bottoms, then simplify. A whole number is itself over 1 ($6 = 6/1$). A mixed number turns improper first ($2 \frac{1}{2} = 5/2$).

Example: $4/5 \times 3/8 = 12/40 = 3/10$. Check that matters: $2/3 \times 3/2 = 6/6 = 1$. A number times its reciprocal is 1.

Negatives: think owe and have

Once every subtraction is 'add the opposite', you are only ever adding, so you do not need a page of sign rules. Positive is money you have; negative is money you owe.

$-3 + (-4)$: owe 3 and owe 4 is owe 7, so -7 . $5 + (-2)$: have 5, owe 2, so 3. $-8 + 3$: owe 8, have 3, still owe 5, so -5 .

Same signs: add the amounts, keep the sign. **Different signs:** take the difference, keep the sign of the bigger pile.

The invisible 1s

Because 1 is the do-nothing number for multiplying, it can hide. Writing it in is the second half of Step One.

x means $1x$, so $x + 2x = 1x + 2x = 3x$, and $5x - x = 5x + (-1x) = 4x$ (not 5). x means x^1 , so x times $x^2 = x^1$ times $x^2 = x^3$ (add the exponents), not x^2 .

Step One, every time: translate each subtraction to adding the opposite and each division to multiplying by the reciprocal, then write in the invisible 1s. Now the whole expression is addition and multiplication.

Worked examples

- $9 - 4 = 9 + (-4) = 5$.
- $12 / 3 = 12 \times 1/3 = 12/3 = 4$.
- $6 / (3/4) = 6/1 \times 4/3 = 24/3 = 8$.
- $-6 + (-5) = -11$ (owe and owe).
- $5x - x = 5x + (-1x) = 4x$.
- x times $x^2 = x^1$ times $x^2 = x^3$.

Practice

A. Translate each subtraction to addition, then evaluate.

- 1) $8 - 5$
- 2) $3 - 10$
- 3) $x - 6$

B. Translate each division to multiplication, then evaluate the number ones.

- 4) $20 / 4$
- 5) $15 / (5/2)$
- 6) $x / 3$

C. Multiply the fractions and simplify.

- 7) $2/5 \times 3/4$
- 8) $5/6 \times 3/5$
- 9) $2/3 \times 3/2$

D. Add, thinking owe and have.

- 10) $-4 + (-9)$
- 11) $-12 + 7$
- 12) $6 + (-15)$

E. Write in the invisible 1s, then simplify.

- 13) $x + 4x$
- 14) $7x - x$
- 15) x^2 times x

F. Do Step One on the whole expression, then find the value.

16) $5 - 12 \div 4$

Answer key

- A. 1) $8 + (-5) = 3$ 2) $3 + (-10) = -7$ 3) $x + (-6)$
B. 4) $20 \times \frac{1}{4} = 5$ 5) $15 \times \frac{2}{5} = \frac{30}{5} = 6$ 6) $x \times \frac{1}{3}$
C. 7) $\frac{6}{20} = \frac{3}{10}$ 8) $\frac{15}{30} = \frac{1}{2}$ 9) $\frac{6}{6} = 1$
D. 10) -13 11) -5 12) -9
E. 13) $5x$ 14) $6x$ 15) x^3
F. 16) $5 + (-(12 \times \frac{1}{4})) = 5 + (-3) = 2$