Rates with Fractions

Students take apart the words unit, rate, and per, then find unit rates when both amounts are fractions with the reciprocal method (multiply the per amount by its reciprocal to get 1, and the other amount by the same number), read a complex fraction as the same move, and choose which of a ratio's two unit rates answers a question.

TIME	25 minutes (one student with a parent or tutor)
WHO	7th grade, about age 12. Opens Unit 1 and the Math 7 course.
MATERIALS	Student worksheet (PDF), the Deep Dive 5 page for this lesson, pencil.
PRIOR KNOWLEDGE	Unit rates with whole numbers, reciprocals, and multiplying fractions (6th grade, NC.6.RP.2 and NC.6.NS.1).
STANDARDS	NC.7.RP.1.
KEEP OUT	The words proportional and constant of proportionality (Lessons 2 and 3). Percent (Unit 2).

Learning goals

1. Say what unit, rate, and per mean, and that a unit rate is how much for one.
2. Find a unit rate when both amounts are fractions: multiply the per amount by its reciprocal to get 1, and the other amount by the same number.
3. Read a complex fraction as the same move: multiply top and bottom by the reciprocal of the bottom.
4. Choose which of the two unit rates answers a question.

Vocabulary: say both, every time

The rule for the whole lesson: whenever the adult or the student says a math word, they follow it with the plain-English version. Proper term first, plain English second, until the student does it without being asked.

TERM	LOOKS LIKE	PLAIN ENGLISH
Unit rate	2 miles per hour	How much for one.
Per	miles per hour	For each.
Reciprocal	$\frac{1}{4}$ and 4	The flip of a fraction; the two multiply to 1.
Complex fraction	$\frac{\frac{1}{2}}{\frac{1}{4}}$	A fraction over a fraction; it means divide.
Ratio	$\frac{1}{2}$ mile to $\frac{1}{4}$ hour	A comparison of two amounts.

> THE ONE SENTENCE TO SAY OUT LOUD
>
> What do I multiply the time by to get 1? Its reciprocal. Then multiply the distance by the same number.

Lesson timeline (25 minutes)

0:00 to 0:04	Warm-up: whole numbers. 120 miles in 2 hours. Ask: what did you do to get 60? Why? Then take apart unit, rate, per.
0:04 to 0:12	Mini-lesson: Julia. Ask: what do I multiply a quarter hour by to get 1 hour? Four. The reciprocal. Multiply the miles by 4 too. The picture (four quarter hours, a half mile each) confirms it. Then show the same move inside the complex fraction.
0:12 to 0:16	Two unit rates. Make the hours 1, or make the miles 1. Which one answers which question? The $1\frac{1}{3}$ hours and 5.2 miles examples.
0:16 to 0:22	Practice. The which-rate questions and the practice problems, or Worksheet Parts A to C. Same buttons: Hint, Show next step, Reveal answer, Hide help.
0:22 to 0:25	Exit ticket. Worksheet Part D. Success looks like 3 miles per hour with the division shown, and per means for each.

Misconceptions to listen for

- **Making the wrong amount into 1.** The amount after per becomes 1: miles per hour means the hours become 1.
- **Multiplying only one amount.** Both get the same multiplier, or the ratio changes.
- **Dropping the units.** 2 is not a rate. 2 miles per hour is.
- **A complex fraction is a new kind of thing.** It is the same move: the bottom times its reciprocal becomes 1.

Support and stretch

Support

Stay with the picture: draw the whole (an hour, a wall) split into the fraction pieces and count how many pieces. That count is the reciprocal. Multiply both amounts by it. Only then show the complex fraction, and match it to the picture.

Stretch

Ask for a rate where both amounts are mixed numbers ($2\frac{1}{2}$ miles in $\frac{3}{4}$ hour) and both unit rates for it. Ask which unit rate a bus schedule would print, and why.

Where this goes next

Lesson 2: what makes a relationship proportional. The unit rate from this lesson becomes the number every row of a table divides to.

> THE HABIT TO BUILD
>
> Ask: what do I multiply by to get 1? Then give the other amount the same number.