7TH GRADE MATH · STUDY GUIDE

Fractional Unit Rates & Proportionality

Everything for Thursday's test, with clear step-by-step examples and practice problems.

Covers: dividing fractions • fractional unit rates • constant of proportionality from tables

★ THE METHOD: 4 STEPS FOR EVERY RATE PROBLEM

The big idea: **"What do I multiply the time by to get 1? Then multiply the distance (or pages, gallons, etc.) by that same number."** That gives the amount for 1 unit of time, which is the rate.

1. **Turn every mixed number into an improper fraction.**
2. **Ask: "What do I multiply the time by to get 1?"** That number is the **reciprocal** of the time (the time flipped over).
3. **Multiply the distance by that same number.** Now you have the amount for **1** unit of time, which is the unit rate.
4. **Read the question again.** What exactly is it asking, and what units does it want the answer in?

1 First, Let's Review What We'll Use

Two ideas power almost every problem on this test. We'll lock them in now, then use them everywhere. (Her teacher said dividing fractions is a **big part** of the test, and it is built from these exact two ideas.)

REVIEW 1: THE RECIPROCAL, THE NUMBER THAT TURNS SOMETHING INTO 1

The **reciprocal** of a fraction is that fraction flipped over. The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

Why we care: a fraction times its reciprocal **always equals 1**: $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$. So whenever you need to turn a number *into 1*, multiply it by its reciprocal.

Works for any number: the whole number 5 = $\frac{5}{1}$, so its reciprocal is $\frac{1}{5}$. Turn mixed numbers improper first: $2\frac{3}{4} = \frac{11}{4}$, reciprocal $\frac{4}{11}$.

REVIEW 2: "DIVIDED BY" AND "OVER" ARE THE SAME THING

x ÷ y means the same as **x over y**, the fraction $\frac{x}{y}$. Always. "6 ÷ 2" is just "$\frac{6}{2}$."

This stays true even when x and y are *themselves* fractions. Then you get a fraction over a fraction, called a **complex fraction**:

$$\frac{2}{3} \div \frac{3}{4} \quad \text{is the same as} \quad \frac{\frac{2}{3}}{\frac{3}{4}}$$

> *"I want the **denominator (bottom) to become 1**, because anything over 1 is just itself. The bottom is $\frac{3}{4}$, so I multiply by its **reciprocal**, $\frac{4}{3}$. To keep the value the same, I multiply the top by $\frac{4}{3}$ too. That is the same as multiplying the whole thing by $\frac{4}{3}$ **over** $\frac{4}{3}$, which equals 1."*

$$\frac{\frac{2}{3}}{\frac{3}{4}} \times \frac{\frac{4}{3}}{\frac{4}{3}} = \frac{\frac{2}{3} \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}} = \frac{\frac{8}{9}}{1} = \frac{8}{9}$$

The denominator (bottom) became 1 ($\frac{3}{4} \times \frac{4}{3} = 1$). Anything over 1 is just itself, so **we don't write the "over 1".** We are simply left with $\frac{8}{9}$.

✎ REVIEW 3: THE SHORTCUT IS "KEEP, CHANGE, FLIP"

Look at what just happened. We **kept** the numerator (top), and multiplied by the **reciprocal** (the "flip") of the denominator (bottom). The denominator did its job of turning into 1 and dropped away. That is *all* "keep, change, flip" ever was, and now you know why it works:

Dividing by a fraction = multiplying by its reciprocal.

You may use that shortcut from here on, because you understand where it comes from.

Your Turn: Review Practice

R1. Write the reciprocal of each: (a) $\frac{4}{7}$ (b) 9 (c) $1\frac{3}{5}$

R2. Show that $\frac{3}{4}$ times its reciprocal equals 1.

R3. Write it as a complex fraction, then multiply by the reciprocal of the bottom over itself to make the denominator (bottom) 1: $\frac{4}{5} \div \frac{2}{3}$

R4. Same steps: $\frac{5}{6} \div \frac{1}{2}$

R5. Same steps (make the mixed number improper first): $2\frac{1}{4} \div \frac{3}{8}$

◆ THIS IS WHY THE METHOD WORKS

When you multiply the time by "what makes it 1," the time turns into **1** and drops away (just like the denominator did above). That is why, to find a rate, you only have to multiply the *distance* by that same number. That is Steps 2 and 3 of the method, and we use it on every problem from here on.

2 The Method in Action

Let's walk the 4 steps on a real problem. Say each step out loud as you go.

✎ WORKED EXAMPLE

Mr. X walked $7\frac{1}{2}$ miles in $2\frac{3}{4}$ hours. How many miles per hour?

1. **Improper fractions.** $7\frac{1}{2} = \frac{15}{2}$ miles, and $2\frac{3}{4} = \frac{11}{4}$ hours.
2. **What do I multiply the time by to get 1?** The time is $\frac{11}{4}$, so I multiply by its reciprocal, $\frac{4}{11}$. (Check: $\frac{11}{4} \times \frac{4}{11} = 1$ ✓)
3. **Multiply the distance by that same $\frac{4}{11}$.** $\frac{15}{2} \times \frac{4}{11} = \frac{60}{22} = \frac{30}{11} = 2\frac{8}{11}$ miles. That is the distance for **1** hour, so the rate is **$2\frac{8}{11}$ miles per hour** (about 2.7 mph).
4. **Reread the question.** It asks for **miles per hour**, and our answer is in miles per hour. ✓ Done.

Notice you never had to worry about "which number goes on top." You make the time into 1, then multiply the distance by that same number.

3 Backup Trick: Rewrite It With Easy Numbers

The 4-step method usually makes the setup clear (the time goes in Step 2, the distance in Step 3). But if a problem's wording ever confuses you, rewrite the *exact same* problem with simple whole numbers to see what to do, then do the same thing to the real numbers.

◆ **STRATEGY: SWAP IN EASY NUMBERS**

Real problem: Maria paints $\frac{2}{3}$ of a wall in $\frac{3}{4}$ of an hour. How much wall per hour? *(Do I divide wall by time, or time by wall??)*

1. **Rewrite with easy numbers:** "Maria paints **6 walls** in **2 hours**. How much per hour?"
2. That one's easy: obviously **6 ÷ 2 = 3 walls per hour**. So the move is **wall ÷ time**.
3. Do the *same operation* with the real numbers: $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$
4. Answer: **$\frac{8}{9}$ of the wall per hour.**

Golden rule: the easy-number version tells you the *plan*. The fractions just follow the same plan.

4 Who's Faster? Comparing Rates

To compare, get **everyone's** rate in the **same** unit (like miles per 1 hour). Then just compare the numbers.

✎ **WORKED EXAMPLE**

Mr. Alvarez walked $4\frac{1}{2}$ miles in $1\frac{1}{2}$ hours. Mr. Boone walked $6\frac{2}{3}$ miles in 2 hours. Who walked faster?

1. **Alvarez**, miles per hour: $\frac{9}{2} \div \frac{3}{2} = \frac{9}{2} \times \frac{2}{3} = \frac{18}{6} =$ **3 mph**
2. **Boone**, miles per hour: $\frac{20}{3} \div 2 = \frac{20}{3} \times \frac{1}{2} = \frac{20}{6} =$ **$3\frac{1}{3}$ mph**
3. Compare: $3\frac{1}{3} > 3$, so **Mr. Boone walked faster.**

5 Step 4, Part 1: What Are They Really Asking?

Sometimes the question wants a *different* amount of time than the rate you found. Do Steps 1 to 3 to get the rate for **1**, then in Step 4 scale up to what they asked for.

✎ **WORKED EXAMPLE**

Bob reads $6\frac{2}{3}$ pages in $\frac{4}{5}$ of an hour. How many pages in 3 hours?

1. **Improper fractions.** $6\frac{2}{3} = \frac{20}{3}$ pages, $\frac{4}{5}$ hour.
2. **What times the time gives 1?** The time is $\frac{4}{5}$, so multiply by its reciprocal, $\frac{5}{4}$.
3. **Multiply the pages by that same $\frac{5}{4}$.** $\frac{20}{3} \times \frac{5}{4} = \frac{100}{12} = \frac{25}{3} =$ **$8\frac{1}{3}$ pages in 1 hour.**
4. **Reread: they want 3 hours, not 1.** So multiply the 1-hour rate by 3: $8\frac{1}{3} \times 3 = \frac{25}{3} \times 3 =$ **25 pages.**

Find the "per 1" first, *then* read Step 4 to see how much time they actually want.

6 Constant of Proportionality From a Table

The "constant of proportionality" (called **k**) is just the unit rate hiding in a table. It's the **y ÷ x** for every row, and in a proportional table, every row gives the *same* answer.

THE RULE

k = $\frac{y}{x}$ (output ÷ input). If every row gives the same **k**, the table is proportional and the equation is **y = kx**.

✎ WORKED EXAMPLE: FIND K

Distance walked over time:

Time (hours), x	$\frac{1}{2}$	$1\frac{1}{2}$	3
Distance (miles), y	$1\frac{1}{4}$	$3\frac{3}{4}$	$7\frac{1}{2}$

1. Do y ÷ x for a row: $1\frac{1}{4} \div \frac{1}{2} = \frac{5}{4} \times \frac{2}{1} = \frac{10}{4} = \mathbf{2\frac{1}{2}}$
2. Check another row: $7\frac{1}{2} \div 3 = \frac{15}{2} \times \frac{1}{3} = \frac{15}{6} = \mathbf{2\frac{1}{2}}$ ✓ same!
3. So **k = $2\frac{1}{2}$** miles per hour, and the equation is **y = $2\frac{1}{2}$x.**

⚠ IS IT EVEN PROPORTIONAL? CHECK EVERY ROW.

x	1	2	3
y	2	5	8

y ÷ x: $\frac{2}{1}$=2, $\frac{5}{2}$=2.5, $\frac{8}{3}$≈2.67. The answers are **different**, so this table is **NOT proportional**. There's no single **k**.

7 Step 4, Part 2: What Units Do They Want?

The other half of Step 4: the numbers might be in one unit while the question asks for another. After you find the rate, make sure it is in the units they asked for.

⚠ WORKED EXAMPLE

Sam runs $\frac{1}{3}$ mile in 4 minutes. At this rate, how far does he run in 1 hour?

> *"The numbers are in **minutes**, but the question says **hour**. Those don't match, so I need to convert. 1 hour = 60 minutes."*

1. **Rate for 1 minute.** The time is 4 minutes, so multiply the distance by its reciprocal, $\frac{1}{4}$: $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ mile per minute.
2. In 1 hour (60 minutes): $\frac{1}{12} \times 60 = \frac{60}{12} =$ **5 miles.**

Common unit swaps to watch for: minutes ↔ hours (×60 or ÷60), feet ↔ yards (3 ft = 1 yd), inches ↔ feet (12 in = 1 ft), cups ↔ pints (2 cups = 1 pint).

Practice Problems

Show your work in the space under each problem. Answer key is on the last pages.

Part A: Dividing Fractions WARM-UP

1. $\frac{5}{6} \div \frac{2}{3} =$ _____

2. $3\frac{1}{3} \div \frac{5}{6} =$ _____

3. $\frac{7}{8} \div 1\frac{3}{4} =$ _____

Part B: Fractional Unit Rates

4. Mr. Diaz walked $3\frac{1}{4}$ miles in $\frac{1}{2}$ hour. How many miles per hour?

5. A snail moves $\frac{2}{5}$ foot in $\frac{2}{3}$ minute. How many feet per minute?

6. Maria fills $\frac{3}{4}$ of a page in $\frac{1}{2}$ a minute. *(Try "easy numbers" if you're stuck: which do you divide?)* How much of a page per minute?

Part C: Who's Faster?

7. Runner A jogs $2\frac{1}{2}$ miles in $\frac{1}{2}$ hour. Runner B jogs $4\frac{2}{3}$ miles in 1 hour. Who is faster? Show both rates in miles per hour.

Part D: How Many in ___? (Steps)

8. A printer prints $5\frac{1}{4}$ pages in $\frac{3}{4}$ minute. How many pages in 4 minutes? (Step 1: per minute. Step 2: × 4.)

Part E: Constant of Proportionality

9. Find **k** and write the equation y = **k**x.

x	2	4	6
y	3	6	9

10. Is this table proportional? Explain how you know.

x	1	2	3
y	2	5	8

Part F: Watch the Units

11. A faucet fills $\frac{3}{4}$ gallon in $\frac{1}{2}$ minute. How many gallons in 1 hour? *(Careful: minutes vs. hour!)*

Answer Key: With Worked Steps

For the tutor / parent. Each answer shows the thinking, not just the number.

Review Practice

R1. (a) reciprocal of $\frac{4}{7}$ is $\frac{7}{4}$ (b) $9 = \frac{9}{1}$, reciprocal $\frac{1}{9}$ (c) $1\frac{3}{5} = \frac{8}{5}$, reciprocal $\frac{5}{8}$

R2. $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = \mathbf{1}$ ✓

R3. $\dfrac{\frac{4}{5}}{\frac{2}{3}} \times \dfrac{\frac{3}{2}}{\frac{3}{2}} = \dfrac{\frac{4}{5} \times \frac{3}{2}}{1} = \frac{12}{10} = \mathbf{1\frac{1}{5}}$

R4. $\frac{5}{6} \times \frac{2}{1} = \frac{10}{6} = \mathbf{1\frac{2}{3}}$

R5. $2\frac{1}{4} = \frac{9}{4}$; $\frac{9}{4} \times \frac{8}{3} = \frac{72}{12} = \mathbf{6}$

Part A: Dividing Fractions

1. $\frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{15}{12} = \mathbf{1\frac{1}{4}}$

2. $3\frac{1}{3} = \frac{10}{3}$; $\frac{10}{3} \times \frac{6}{5} = \frac{60}{15} = \mathbf{4}$

3. $1\frac{3}{4} = \frac{7}{4}$; $\frac{7}{8} \times \frac{4}{7} = \frac{28}{56} = \mathbf{\frac{1}{2}}$

Part B: Fractional Unit Rates

4. $\frac{13}{4} \div \frac{1}{2} = \frac{13}{4} \times \frac{2}{1} = \frac{26}{4} = \mathbf{6\frac{1}{2}}$ **miles per hour**

5. $\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2} = \frac{6}{10} = \mathbf{\frac{3}{5}}$ **foot per minute**

6. Easy numbers: "3 pages in 1 minute" → page ÷ time. So $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \mathbf{1\frac{1}{2}}$ **pages per minute**

Part C: Who's Faster?

7. A: $\frac{5}{2} \div \frac{1}{2} = \frac{5}{2} \times \frac{2}{1} =$ **5 mph**. B: $4\frac{2}{3} \div 1 =$ **$4\frac{2}{3}$ mph**. $5 > 4\frac{2}{3}$, so **Runner A is faster.**

Part D: How Many in ___?

8. Step 1 (per minute): $5\frac{1}{4} \div \frac{3}{4} = \frac{21}{4} \times \frac{4}{3} = \frac{84}{12} =$ **7 pages/min**. Step 2: 7 × 4 = **28 pages.**

Part E: Constant of Proportionality

9. **k** = y ÷ x = $\frac{3}{2} = 1\frac{1}{2}$ (check: $\frac{6}{4}=1\frac{1}{2}$, $\frac{9}{6}=1\frac{1}{2}$ ✓). Equation: **$y = 1\frac{1}{2}x$.**

10. y ÷ x: $\frac{2}{1}=2$, $\frac{5}{2}=2.5$, $\frac{8}{3}\approx 2.67$. Not equal, so **NOT proportional.**

Part F: Watch the Units

11. Per minute: $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = 1\frac{1}{2}$ gal/min. In 1 hour: $1\frac{1}{2} \times 60 =$ **90 gallons.**
