Discovering Properties with a Cayley Table

Exploring patterns in a special math operation

Name: ____________________ Date: ____________________

📋 Your Cayley Table Reference

⊕	Black	White	Yellow	Pink	Orange	Green
Black	Black	White	Yellow	Pink	Orange	Green
White	White	Yellow	Pink	Orange	Green	Black
Yellow	Yellow	Pink	Orange	Green	Black	White
Pink	Pink	Orange	Green	Black	White	Yellow
Orange	Orange	Green	Black	White	Yellow	Pink
Green	Green	Black	White	Yellow	Pink	Orange

Black White Yellow Pink Orange Green

Part 1: Warm-Up — Using the Table

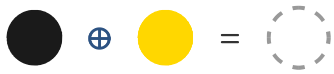
How to use the table: Find the first color in the left column, then find the second color in the top row. The answer is where they meet!

1. Yellow ⊕ Pink = ◌
2. Orange ⊕ Green = ◌
3. Green ⊕ White = ◌
4. White ⊕ White = ◌
5. Pink ⊕ Orange = ◌
6. Yellow ⊕ Orange = ◌

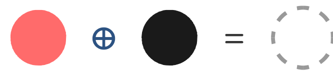
Part 2: The Identity Property

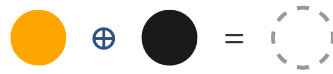
> The **identity element** is a special color that doesn't change anything! When you combine it with any color, you get that same color back. It's like adding 0 or multiplying by 1.

Try combining Black with different colors:

7. 

8. 

9. 

10. 

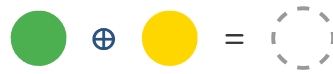
🔍 Discovery Questions

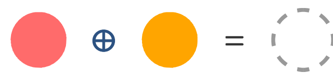
11. Which color is the identity element? 

12. How do you know? What pattern did you notice?

Part 3: The Commutative Property

A math operation is **commutative** if you can switch the order and still get the same answer. For example, 3 + 5 = 5 + 3. Let's see if our ⊕ operation works the same way!

13a.  13b.  Same? _____

14a.  14b.  Same? _____

15a.  15b.  Same? _____

🔍 **Discovery Questions**

16. Is the ⊕ operation commutative? (Circle one): **YES** / **NO**

17. Look at the Cayley table. What do you notice if you imagine folding it along the diagonal (from the ⊕ symbol to the bottom-right corner)?

Part 4: The Associative Property

> A math operation is **associative** if it doesn't matter how you group things. For example, (2 + 3) + 4 = 2 + (3 + 4). Let's test if our ⊕ operation works this way!

Test 1: Does (⚪ ⊕ 🟡) ⊕ 🔴 equal ⚪ ⊕ (🟡 ⊕ 🔴)?

LEFT side: Do the parentheses first

18. First, find ⚪ ⊕ 🟡 = ____

19. Then, find (your answer) ⊕ 🔴 = ____ ← This is the LEFT side result

RIGHT side: Different grouping

20. First, find 🟡 ⊕ 🔴 = ____

21. Then, find ⚪ ⊕ (your answer) = ____ ← This is the RIGHT side result

Test 2: Try another! Does (🟠 ⊕ 🟢) ⊕ 🟡 equal 🟠 ⊕ (🟢 ⊕ 🟡)?

22. LEFT side: (🟠 ⊕ 🟢) ⊕ 🟡 = ____ ⊕ 🟡 = ____

23. RIGHT side: 🟠 ⊕ (🟢 ⊕ 🟡) = 🟠 ⊕ ____ = ____

🔍 Discovery Questions

24. Is the ⊕ operation associative? (Circle one): **YES** / **NO**

25. Why does this matter? When an operation is associative, we don't need parentheses! Explain in your own words why that's useful:

Part 5: Inverses — The "Undo" Color

Every color has a special partner called its **inverse**. When you combine a color with its inverse, you always get back to the identity (Black). It's like having an "undo button" for each color!

Find the inverse for each color — the color that brings you back to Black:

26. White ⊕ ◌ = Black White's inverse is: ____________

27. Yellow ⊕ ◌ = Black Yellow's inverse is: ____________

28. Pink ⊕ ◌ = Black Pink's inverse is: ____________

29. Orange ⊕ ◌ = Black Orange's inverse is: ____________

30. Green ⊕ ◌ = Black Green's inverse is: ____________

31. Black ⊕ ◌ = Black Black's inverse is: ____________

🔍 Discovery Questions

32. Look at your answers. Which color is its own inverse? Why do you think that happens?

33. Do you notice any pairs of colors that are inverses of each other? List them:

Part 6: Solving Equations with Inverses

Now you can solve equations! If you want to find the mystery color in ⊕ ? = , you can "undo" the Pink by combining both sides with Pink's inverse!

Example: Solve ⊕ ? =

Think: I need to "undo" White. White's inverse is Green.
Apply inverse to both sides: Green ⊕ (White ⊕ ?) = Green ⊕ Orange
Left side simplifies: Since Green ⊕ White = Black, and Black ⊕ ? = ?, we get just ?
Right side: Green ⊕ Orange = Pink
Answer: ? = Pink ✓

Now you try! Find the mystery color:

34. ⊕ ? =

Hint: What is Yellow's inverse? Use it to undo Yellow.

Mystery color:

35. ⊕ ? =

Mystery color:

36. ⊕ ? =

Hint: What color doesn't change Orange?

Mystery color:

37. ? ⊕ ● = ●

Hint: What color combined with Green gives Black?

Mystery color:

⭐ Bonus Challenge

38. **Solve this two-step problem:** ○ ⊕ (● ⊕ ?) = ●

Hint: First undo White, then undo Yellow.

Mystery color:

39.

Big Picture Question:

You've discovered four important properties today: Identity, Commutativity, Associativity, and Inverses. Why do you think mathematicians care about these properties? How might they help make math easier?