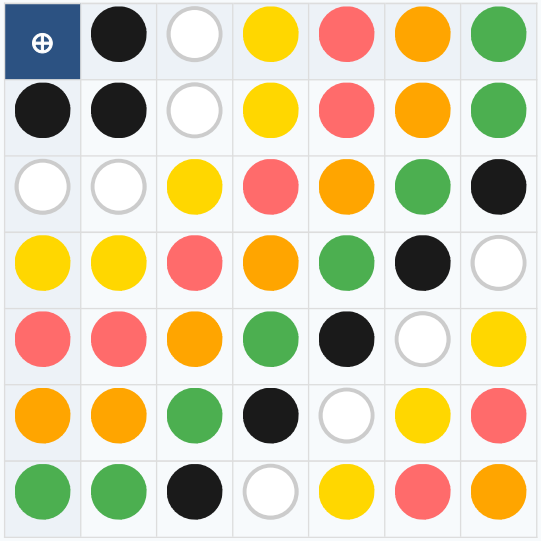
Discovering Properties with Two Operations

How ⊕ and ★ work together

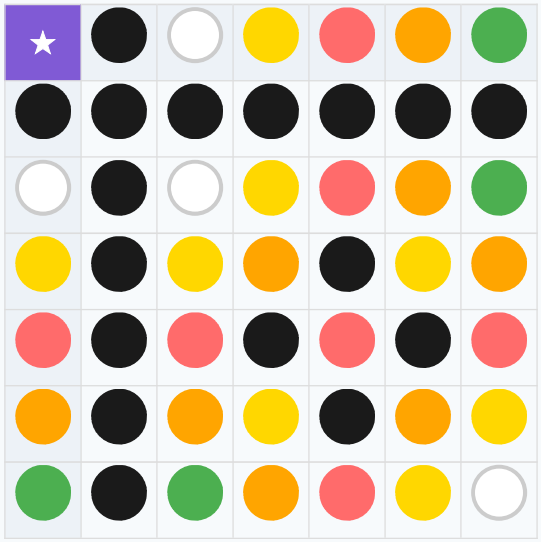
Name: _____ Date: _____

📋 Your Two Cayley Tables

⊕ "Add" Table



★ "Multiply" Table



Black White Yellow Pink Orange Green

⚠️ **Important:** Be careful to use the correct table! Use the ⊕ table for ⊕ problems and the ★ table for ★ problems.

Part 1: Warm-Up — Using Both Tables

> **Remember:** Find the first color in the left column, then find the second color in the top row. The answer is where they meet. Use the RIGHT table for each symbol!

Using the ⊕ table:

1. ○ ⊕ ● = ◌
2. ● ⊕ ● = ◌
3. ● ⊕ ● = ◌

Using the ★ table:

4. ○ ★ ● = ◌
5. ● ★ ● = ◌
6. ● ★ ● = ◌

🔍 **Notice Something?**

7. Compare your answers for #1 and #4 (both started with White ⊕ or ★ Yellow). Did you get the same result? Why or why not?

Part 2: Two Different Identities!

> Each operation has its own identity element — a color that doesn't change anything when you use it with that operation. Can you find which color is the identity for each?

Testing ⊕ (adding)

8. ● ⊕ ● = ◌

9. ● ⊕ ● = ◌

Testing ★ (multiplying)

10. ○ ★ ● = ◌

11. ● ★ ○ = ◌

🔍 Discovery Questions

12. Which color is the identity for ⊕? ◌

13. Which color is the identity for ★? ◌

14. This is just like regular numbers! Fill in the blanks:

Adding _____ doesn't change a number. Multiplying by _____ doesn't change a number.

Part 3: The Zero Property — Black Swallows Everything!

> Something very different happens when you ★ multiply by Black. Look at the Black row in the ★ table. What do you notice?

15.  = ◌

16.  = ◌

17.  = ◌

18.  = ◌

🔍 Discovery Questions

19. What always happens when you ★ multiply anything by Black?

20. Compare how Black behaves in each operation:

- With ⊕: Black is the _____ (doesn't change things)
- With ★: Black _____ (absorbs everything)

This is exactly like zero in regular math! Zero is the identity for addition, but it "absorbs" everything in multiplication (any number × 0 = 0).

Part 4: Are Both Operations Commutative?

> Remember: An operation is **commutative** if switching the order doesn't change the answer. Let's test both operations!

Test ⊕ (adding):

21a.  Same? _____

Test ★ (multiplying):

22a.  Same? _____

23a.  Same? _____

🔍 Discovery Question

24. Are BOTH operations commutative? (Circle): **YES** / **NO**

Part 5: The Distributive Property — Two Operations Working Together!

The **distributive property** connects multiplication and addition. It says you can "distribute" multiplication over addition. Here's the rule:

$$a \star (b \oplus c) = (a \star b) \oplus (a \star c)$$

Test 1: Does 🟡 ★ (⚪ ⊕ 🔴) equal (🟡 ★ ⚪) ⊕ (🟡 ★ 🔴)?

LEFT side: Do the parentheses first, then multiply

25. First: ⚪ ⊕ 🔴 = ◌

26. Then: 🟡 ★ (your answer) = ◌ ← LEFT side result

RIGHT side: Distribute the Yellow, then add

27. Find: 🟡 ★ ⚪ = ◌ and 🟡 ★ 🔴 = ◌

28. Then: (first answer) ⊕ (second answer) = ◌ ← RIGHT side result

29. Do the LEFT and RIGHT sides match? _____

Test 2: Try 🔴 ★ (🟡 ⊕ 🟠)

30. LEFT: 🔴 ★ (🟡 ⊕ 🟠) = 🔴 ★ ◌ = ◌

31. RIGHT: (🔴 ★ 🟡) ⊕ (🔴 ★ 🟠) = ◌ ⊕ ◌ = ◌

🔍 **Discovery Questions**

32. Does the distributive property work for our color operations? (Circle): **YES** / **NO**

33. In regular math, $3 \times (4 + 5) = (3 \times 4) + (3 \times 5)$. Explain in your own words what the distributive property lets you do:

⭐ Bonus Challenge

34. **Use the distributive property** to solve this WITHOUT looking up  in the table:

Hint: What is Yellow ⊕ Orange? Then what happens when you ★ by Green?

My answer:

35.

Multiplicative Inverses:

In the ⊕ table, every color has an inverse (a partner that gives you Black). But in the ★ table, not every color has an inverse! Look at the ★ table and find which colors CAN give you White (the ★ identity).

Hint: Look for White in the ★ table. Which colors can combine to make White?

36.

Big Picture:

Why do you think mathematicians study how TWO operations work together? Think about regular math — how do addition and multiplication connect?

Summary: What You Discovered

Fill in this summary of what you learned:

Identity elements:

- The identity for ⊕ is ◌ (like adding ____ in regular math)
- The identity for ★ is ◌ (like multiplying by ____ in regular math)

The Zero Property:

- Black ★ anything = ◌ (Black absorbs everything in ★, just like ____ in regular multiplication)

Commutativity:

- Both ⊕ and ★ are commutative: order (does / doesn't) matter.

Distributive Property:

- a ★ (b ⊕ c) = (a ★ b) ⊕ (a ★ c) — you can "distribute" the ★ over the ⊕