

Volume City

Strategic Urban Planning Challenge

Student Worksheet Packet

Name: _____

Date Started: _____ **Date Completed:** _____

Your Mission

Congratulations! You've been hired as lead urban planner for a new city development. You have:

- **Budget:** \$125,000
- **Land:** 1,000 km² of flat, prepared property
- **Goal:** House at least 10,000 people with all essential services


The Challenge: Your city must be optimized for efficiency, sustainability, and quality of life!

Success Criteria - Your City Must:

- House at least 10,000 people
- Stay within \$125,000 budget
- Provide emergency services within appropriate distances
- Include parks and public spaces
- Be justified with mathematical evidence

Phase 1: Research & Analysis

Investigation 1A: Population Capacity Research

 **Research Tip:** Look up "population density" and "square feet per person" for different building types. Remember: 1 cm in your model = ~10 meters in real life.

Building Type: HIGH-RISE RESIDENTIAL

Research Notes:

My Calculations:

Average apartment size: _____ square feet

Average residents per apartment: _____ people

Square feet per person: _____ sq ft/person

Estimated people per 100 cm³: _____ people

Building Type: HOSPITAL

Research Notes:

My Calculations:

Hospital beds per floor: _____ beds

Square feet per bed (including space): _____ sq ft

People served per 100 cm³: _____ people

Building Type: SCHOOL

Students per classroom: _____ students

Square feet per student: _____ sq ft

Students per 100 cm³: _____ students

Investigation 1B: Service Radius Requirements

Research Question 1: Fire Station Response Distance

Recommended maximum distance in real cities: _____ miles

Why this matters:

Converting to my model grid (1 grid unit = _____ meters):

Maximum distance on my grid: _____ units

Research Question 2: Distance Calculation Method

I will use (check one):

- Straight-line distance
- City-block distance (recommended for cities with roads)

Reason for my choice:

Example calculation showing my distance method:

Building A is at grid position (3, 5) Building B is at grid position (7, 9) Distance = _____ units Show your work:

Investigation 1C: Scaling Relationships

 **Investigation Goal:** Discover what happens to volume when you change dimensions!

Original building: $4\text{ cm} \times 3\text{ cm} \times 10\text{ cm}$

Question	Dimensions	Volume	Factor
1. Original volume	$4 \times 3 \times 10$	_____ cm^3	—
2. Double ONLY height	$4 \times 3 \times 20$	_____ cm^3	\times _____
3. Double ALL dimensions	$8 \times 6 \times 20$	_____ cm^3	\times _____

 **Critical Thinking:** When you double all three dimensions, why doesn't the volume just double? What pattern do you notice?

Pattern I discovered:

When you double all dimensions of ANY rectangular prism, volume increases by a factor of: _____

This happens because:

Investigation 1D: Optimization Challenge

Goal: Create a hospital with EXACTLY 720 cm³ of space

Option	Length	Width	Height	Volume	Surface Area
Option 1	_____	_____	_____	720 cm ³	_____ cm ²
Option 2	_____	_____	_____	720 cm ³	_____ cm ²
Option 3	_____	_____	_____	720 cm ³	_____ cm ²

Surface Area Formula: $SA = 2lw + 2lh + 2wh$

Surface area matters because less surface = less building materials = lower real construction cost!

Show your surface area calculations for Option 1:

$$SA = 2(\text{___} \times \text{___}) + 2(\text{___} \times \text{___}) + 2(\text{___} \times \text{___}) \quad SA = \text{___} + \text{___} + \text{___} = \text{___} \text{ cm}^2$$

Analysis:

Which option has the smallest surface area? **Option** _____

What shape tends to be most efficient (least surface area for a given volume)?

Cube-like shapes Very tall, thin Very flat, wide

Explain your reasoning:

Investigation 1E: Budget Strategy Challenge

Current Situation: You're \$15,000 OVER budget with these buildings planned:

Building	Dimensions	Volume	Cost
Hospital	$6 \times 4 \times 15$	360 cm^3	\$21,600
High-Rise	$5 \times 5 \times 18$	450 cm^3	\$27,000
School	$8 \times 6 \times 12$	576 cm^3	\$28,800
Fire Station	$5 \times 4 \times 10$	200 cm^3	\$10,000

STRATEGY 1: I would modify _____

Original dimensions: _____ \times _____ \times _____ = _____ cm^3 = \$ _____

New dimensions: _____ \times _____ \times _____ = _____ cm^3 = \$ _____

Savings: \$ _____

Impact on functionality:

STRATEGY 2: I would modify _____

Original dimensions: _____ \times _____ \times _____ = _____ cm^3 = \$ _____

New dimensions: _____ \times _____ \times _____ = _____ cm^3 = \$ _____

Savings: \$ _____

BEST STRATEGY: My recommended approach is Strategy _____ because:

Phase 2: Strategic Design

Your Task: Create THREE different city designs using different strategies.

Each design must:

- House at least 10,000 people
- Stay within \$125,000 budget
- Include all required buildings
- Be drawn on grid paper

Design A: "Skyline City" (Tall Buildings)

Strategy: Build fewer, taller structures to save ground space

Building	L × W × H	Volume	Cost per cm ³	Total Cost
Hospital	___ × ___ × ___	_____ cm ³	\$60	\$_____
High-Rise #1	___ × ___ × ___	_____ cm ³	\$60	\$_____
High-Rise #2	___ × ___ × ___	_____ cm ³	\$60	\$_____
Fire Station	___ × ___ × ___	_____ cm ³	\$50	\$_____
Police Station	___ × ___ × ___	_____ cm ³	\$50	\$_____
City Hall	___ × ___ × ___	_____ cm ³	\$50	\$_____
School	___ × ___ × ___	_____ cm ³	\$50	\$_____
Shopping Mall	___ × ___ × ___	_____ cm ³	\$40	\$_____
Parks	Total: _____ cm ²		\$10/cm ²	\$_____

Roads	Total: _____ cm ²	\$10/cm ²	\$ _____
TOTAL COST			\$ _____

Under/Over Budget: \$ _____ (under/over)

Population Capacity Calculation:

High-Rise #1: _____ cm³ × _____ people/100cm³ = _____ people

High-Rise #2: _____ cm³ × _____ people/100cm³ = _____ people

Total Population: _____ **people**

Meets 10,000 minimum? Yes No

Advantages of this design:

Disadvantages of this design:

Why this design is superior:

Mathematical reasoning behind my choices:

Phase 3: Comparative Analysis

Comparison Matrix

Metric	Design A (Skyline)	Design B (Sprawl)	Design C (Balanced)
Total Cost	\$ _____	\$ _____	\$ _____
Under/Over Budget	\$ _____	\$ _____	\$ _____
Population Capacity	_____ people	_____ people	_____ people
Meets 10,000 minimum?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Total Park Space	_____ cm ²	_____ cm ²	_____ cm ²
Park Space per Person	_____ cm ² / person	_____ cm ² / person	_____ cm ² /person
Efficiency Ratio (Pop. ÷ Cost per \$1000)	_____ people/ \$1000	_____ people/ \$1000	_____ people/ \$1000

How to calculate Efficiency Ratio:

Population ÷ (Total Cost ÷ 1000) = People per \$1000 spent

Example: 12,000 people, \$120,000 cost → 12,000 ÷ 120 = 100 people/\$1000

Final Justification

1. MOST EFFECTIVE DESIGN

The most effective design is Design _____ because:

Specific evidence from my data:

-

-

-

2. TRADE-OFFS ANALYSIS

In my best design, I sacrificed:

In order to gain:

This trade-off was worth it because:

3. EXPANSION PLANNING

If my city must grow by 50% (from 10,000 to 15,000 people):

I would enlarge: _____

Current dimensions: _____ × _____ × _____

New dimensions: _____ × _____ × _____

Why this building:

Mathematical reasoning for my expansion choices:

Mathematical Reflection

1. VOLUME RELATIONSHIPS

When you double all three dimensions of a rectangular prism, the volume increases by a factor of _____.

This happens because:

Hint: Think about the formula $V = l \times w \times h$. What happens when you substitute $2l$, $2w$, and $2h$?

2. OPTIMIZATION DISCOVERY

For a given volume, which shape has the smallest surface area?

Cube-like Very tall Very flat

This matters in construction because:

3. PROPORTIONAL REASONING

If your city's population grows 50%, do you need 50% more of EVERY building?

Yes No

Explain:

4. REAL-WORLD CONNECTION

Research one way real city planners use mathematics similar to this project:

Source:

5. PROBLEM-SOLVING PROCESS

The hardest mathematical decision I made was:

How I worked through it:

Congratulations!

You've completed the Volume City project!

What You Accomplished

You didn't just calculate volumes – you:

- Researched real-world data and converted units
- Discovered the relationship between scaling and volume
- Optimized building designs for efficiency
- Made strategic trade-offs with competing constraints
- Created three different city designs
- Compared designs using mathematical evidence
- Justified your decisions like a real engineer!

This is how mathematicians think!

These problem-solving skills will serve you in any field you pursue.

Volume City: Strategic Urban Planning Challenge

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