Volume City

Teacher Guide & Answer Key

Strategic Urban Planning Challenge

Overview

This guide provides answer keys, teaching strategies, discussion questions, and facilitation tips for the Volume City project. Students work through three phases of mathematical investigation, design, and analysis.

Timeline: 6-8 Hours Over 2-3 Weeks

- **Sessions 1-2:** Phase 1 - Research & Analysis (3-4 hours)
- **Sessions 3-4:** Phase 2 - Strategic Design (3-4 hours)
- **Sessions 5-6:** Phase 3 - Analysis & Reflection (1-2 hours)

Phase 1: Answer Keys & Teaching Strategies

Investigation 1A: Population Capacity Research

💡 **Teaching Tip:** Don't give students the answers! Guide them to research online. Accept reasonable estimates - there's no single "correct" answer.

Sample Answers (student answers will vary):

High-Rise Residential:

- Average apartment: ~800 sq ft
- Residents per apartment: ~2.5 people
- In model scale: ~2-4 people per 100 cm^3

Hospital:

- ~200 sq ft per bed (including space)
- In model: ~20 people served per 100 cm^3

School:

- ~25 students per classroom
- ~35-50 sq ft per student
- In model: ~15-20 students per 100 cm^3

Discussion Questions:

- *"Why might your estimate be different from a classmate's? Is one wrong?"*
- *"What assumptions did you make in your research?"*
- *"How would changing your population estimates affect your city design?"*

Investigation 1B: Service Radius Requirements

Fire Station Response:

- Real cities: ~1.5 miles maximum (5-minute response time)
- If 1 grid unit = 100 meters: ~24 grid units

Distance Calculation Method:

- **Recommended: City-block distance** (more realistic for cities with roads)
- Formula: $|x_2-x_1| + |y_2-y_1|$
- Example: (3,5) to (7,9) = |7-3| + |9-5| = 4 + 4 = 8 units

💡 **Teaching Tip:** For 5th grade, city-block distance is more accessible and actually more realistic than straight-line distance.

Investigation 1C: Scaling Relationships

🎯 **KEY CONCEPT: This is where students discover the cubic relationship! Don't tell them - let them discover it.**

Answers:

Question	Dimensions	Volume	Factor
Original	4 × 3 × 10	120 cm^3	—
Double height only	4 × 3 × 20	240 cm^3	×2
Double all dimensions	8 × 6 × 20	960 cm^3	×8

Key Insight: When you double all dimensions, volume increases by $2^3 = 8$

Why: $V = (2l) \times (2w) \times (2h) = 2\times2\times2 \times (l\times w\times h) = 8V$

Guiding Questions (don't tell - ask!):

- *"What do you notice about the factor by which volume increased?"*
- *"Can you predict what would happen if we tripled all dimensions?" (Answer: 27×)*
- *"Test your prediction with a different starting prism"*

Investigation 1D: Optimization Challenge

Sample Solutions for Volume = 720 cm³:

Option	Dimensions	Surface Area
Option 1	12 × 6 × 10	504 cm²
Option 2	9 × 8 × 10	484 cm²
Option 3	10 × 9 × 8	484 cm² (most cube-like)

Key Discovery: Shapes closer to cubes have LESS surface area for the same volume!

Why This Matters: Less surface area = less materials = lower real construction cost

💡 **Real-World Connection:** This is why shipping containers are nearly cubic, and why bubbles are spherical (spheres have minimum surface area for volume).

Investigation 1E: Budget Strategy Challenge

Current Buildings: $87,400 total (need to cut $15,000)

Sample Strategy 1: Reduce High-Rise height

- Original: 5 × 5 × 18 = 450 cm³ = $27,000
- New: 5 × 5 × 13 = 325 cm³ = $19,500
- Saves: $7,500
- Impact: Houses fewer people but still functional

Sample Strategy 2: Make school more compact

- Original: 8 × 6 × 12 = 576 cm³ = $28,800
- New: 6 × 6 × 12 = 432 cm³ = $21,600
- Saves: $7,200

Best Strategy: Use both approaches for total $14,700 savings

Push for Justification: *Ask "Why did you choose to cut THAT building and not another?" Require quantitative reasoning, not just opinions.*

Phase 2: Design Phase Guidance

Facilitating the Design Process

💡 **Key Principles:**

- Students should physically draw on grid paper (spatial reasoning!)
- This phase takes TIME - multiple hours over several days
- Encourage iteration: "Your first design probably won't be your best"
- Constantly check constraints: budget, population, service radius

Expected Characteristics:

Design Type	Expected Features	Pros/Cons
Design A: Skyline	• Tall buildings (h > l and w) • Fewer total buildings • More vertical growth	+ Saves ground space + More dramatic - Likely more expensive - Access challenges
Design B: Sprawl	• Wide, short buildings • More horizontal spread • Uses more ground space	+ Potentially cheaper + More accessible - Uses more land - Longer distances
Design C: Balanced	• Informed by Phase 1 analysis • Strategic optimization • Cube-like shapes	Should demonstrate mathematical reasoning from research phase

Building Cost Reference

Building Type	Cost per cm^3
Hospital / High-Rise	$60/$cm^3$
Fire Station / Police / City Hall / School	$50/$cm^3$
Shopping Mall	$40/$cm^3$
Parks / Roads	$10/$cm^2$ (area, not volume)

When Students Struggle:

- ***"I don't know what dimensions to use"*** → *"What population do you need? Work backwards."*
- ***"My city is way over budget"*** → *"Which buildings are most expensive per cm^3?"*
- ***"All my designs look the same"*** → *"Make Design A EXTREME - all super tall, then make B opposite."*

Phase 3: Comparative Analysis Guidance

Metrics Explained

1. Total Cost: Sum of all building costs

2. Population Capacity: Volume of residential buildings × people per cm^3

3. Average Distance to Fire Station:

- Calculate distance from fire station to EACH residential building
- Add them up, divide by number of residential buildings

4. Park Space per Person: Total park area ÷ total population

- For comparison: NYC has ~4.9 m^2 per person; Singapore has ~7.8 m^2
- In model: 50 cm^2 per person = ~5 m^2 real-world (good benchmark)

5. Efficiency Ratio: Population ÷ (Cost/1000)

- Example: 12,000 people, $120,000 cost → 12,000 ÷ 120 = 100 people per $1,000
- Higher number = more efficient

Evaluating Student Analysis

Strong Analysis Includes:

- ✅ Specific numbers from comparison matrix
- ✅ Recognition that designs have trade-offs
- ✅ Acknowledgment of competing goals
- ✅ Evidence-based reasoning

Weak Analysis:

- ❌ "Design C is best because I like it"
- ❌ No reference to actual numbers
- ❌ Opinion without justification

Example of Strong Analysis:
"Design B has the lowest cost ($118,000), but Design C is more efficient because it houses 12,000 people versus Design B's 10,500 people. When I calculate the efficiency ratio, Design C gives 104 people per $1000 while Design B only gives 89 people per $1000. Even though Design C costs more total, it's a better value."

Mathematical Reflection - Answer Key

1. Volume Relationships

Answer: When you double all three dimensions, volume increases by a factor of **8**

Why: $V = l \times w \times h \rightarrow$ New volume $= (2l) \times (2w) \times (2h) = 8lwh = 8V$

Extension: If triple all dimensions? $3^3 = 27\times$ volume

2. Optimization Discovery

Answer: Cube-like shapes have smallest surface area for given volume

Why it matters:

- Less materials = lower cost
- Less heat loss = more energy efficient
- Real-world: shipping containers, ice cubes, building design

3. Proportional Reasoning

Answer: NO, you don't need 50% more of every building

Reasoning:

- Fire stations serve areas, not just population (service radius)
- Parks can be shared spaces
- Schools have economies of scale
- One larger school may be more efficient than two small schools

This is sophisticated thinking - celebrate it!

4. Real-World Connection

Sample answers students might find:

- Zoning laws (residential vs. commercial ratios)
- Traffic flow modeling
- Public transit planning
- Emergency response optimization
- Water/electricity infrastructure sizing

Assessment Rubric

Criterion	Exemplary (4)	Proficient (3)	Developing (2)	Beginning (1)
Mathematical Understanding	Deep understanding of volume, scaling, optimization; makes unprompted connections	Accurate calculations; understands basic relationships; adequate explanations	Calculations mostly accurate with guidance; partial understanding	Frequent errors; minimal understanding
Strategic Thinking	Sophisticated trade-off reasoning; choices clearly based on analysis	Generally well-reasoned choices with adequate justification	Some reasoning present but inconsistent	Choices appear arbitrary
Research & Analysis	Thorough research; reasonable estimates; thoughtful constraints	Adequate research with logical estimates	Limited research or unrealistic estimates	Minimal research effort
Communication	Clear, precise math communication with excellent use of evidence	Generally clear with adequate supporting evidence	Unclear or incomplete explanations	Poor communication

Session Planning Guide

Recommended Schedule

Session 1 (60-90 min): Introduction & Research

- Introduce project scenario
- Begin Investigation 1A: Population Research
- Homework: Complete Phase 1 research

Session 2 (60-90 min): Mathematical Investigations

- Investigation 1C: Scaling Relationships (KEY INSIGHT!)
- Investigation 1D: Optimization Challenge
- Investigation 1E: Budget Strategy

Session 3 (90 min): Design A

- Review Phase 1 findings
- Create Design A: "Skyline City"
- Check constraints

Session 4 (90 min): Designs B & C

- Complete Design B: "Sprawl City"
- Begin Design C: "Balanced City"
- Homework: Complete Design C

Session 5 (60-90 min): Comparative Analysis

- Calculate metrics for all three designs
- Complete comparison matrix
- Begin written justification

Session 6 (60 min): Reflection & Extensions

- Discuss final analysis
- Work through reflection questions
- Optional: Bonus challenges or presentation

Quick Reference

When Students Say...

Student Statement	Your Response
"I don't know what to do!"	"Let's look at what you know. Your research told you X about population. Let's work backwards."
"This is too hard!"	"Which part feels hardest? Let's break it into smaller pieces."
"Did I do this right?"	"Tell me your thinking. Why did you choose those dimensions?" (Focus on reasoning, not just answer)
"Can't you just tell me the answer?"	"There isn't one right answer - that's what makes this interesting! Your job is to find a solution that works and explain why."

Key Mathematical Insights

- Scaling: When dimensions multiply by k, volume multiplies by k^3
- Optimization: Cube-like shapes minimize surface area for given volume
- Trade-offs: No perfect solution; must balance competing goals
- Evidence: Mathematical decisions must be justified with data

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