

Discovering Inverses with Cayley Tables

Learning to undo operations and solve equations

Name: _____

Date: _____

Your Cayley Tables Reference

⊕	●	○	●	●	●	●
●	●	○	●	●	●	●
○	○	●	●	●	●	●
●	●	●	●	●	●	○
●	●	●	●	●	○	●
●	●	●	●	○	●	●
●	●	●	○	●	●	●

"Add" Table

★	●	○	●	●	●	●
●	●	○	●	●	●	●
○	●	○	●	●	●	●
●	●	●	●	●	●	●
●	●	●	●	●	●	●
●	●	●	●	●	●	●
●	●	●	●	●	●	●

"Multiply" Table

Part 1: What Is an Inverse?

An inverse is like an "undo button." When you combine something with its inverse, you get back to the identity element—the color that doesn't change anything.

For \oplus , the identity is Black. So we're looking for: color \oplus (inverse) = Black

Find the inverse for each color using the \oplus table:

1. ○ \oplus ○ = ●

2. ● \oplus ○ = ●

3. ● \oplus ○ = ●

4. ● \oplus ○ = ●

5. ● \oplus ○ = ●

6. ● \oplus ○ = ●

Discovery Question

- Does every color have an inverse for \oplus ?
- Which color is its own inverse? How do you know?

Part 2: Solving Equations with \oplus

If you know inverses, you can solve equations without guessing!

To solve: Yellow \oplus ? = Green

Use Yellow's inverse on both sides. Yellow's inverse is Orange (since Yellow \oplus Orange = Black).

Orange \oplus (Yellow \oplus ?) = Orange \oplus Green

This simplifies to: ? = Orange \oplus Green

Guided Example: Solve Yellow \oplus ? = Green

Step 1: Yellow's inverse is Orange (because Yellow \oplus Orange = Black)

Step 2: Add Orange to both sides: Orange \oplus (Yellow \oplus ?) = Orange \oplus Green

Step 3: The left side becomes: (Orange \oplus Yellow) \oplus ? = Black \oplus ? = ?

Step 4: Find Orange \oplus Green in the table: _____

Step 5: So ? = _____

Practice: Solve these equations. Show your work!

9. Pink \oplus ? = Yellow

Inverse I need: _____ Work: _____

Answer:

10. ? \oplus Orange = White

Inverse I need: _____ Work: _____

Answer:

11. Green \oplus ? = Green

Inverse I need: _____ Work: _____

Answer:

Discovery Question

12. When you solve \oplus equations, how many answers do you find? Always one? Sometimes none?

Part 3: Looking for ★ Inverses

Now let's try to find inverses for the ★ operation.
Remember: The identity for ★ is White (not Black!).
So we're looking for: color ★ (inverse) = White

Try to find the ★ inverse for each color:

13.  ★  =  Search the  row. Answer: _____

14.  ★  =  Search the  row. Answer: _____

15.  ★  =  Search the  row. Answer: _____

16.  ★  =  Search the  row. Answer: _____

17.  ★  =  Search the  row. Answer: _____

Discovery Question

18. Did every color have a ★ inverse?

19. Which colors are MISSING their inverse? List them: _____

20. This is surprising! Some colors can't be "undone" with ★. Why might this be?

Part 4: The Annihilation Discovery

Something strange happens with ★. Look at this:

Yellow ★ Pink = ?

Neither Yellow nor Pink is Black... but what do they make together?

Explore these combinations:

21.  ★  = 

22.  ★  = 

23.  ★  = 

24.  ★  = 

Discovery Question

25. What do you notice? Two colors that are NOT Black can multiply to make _____!

26. This is called "annihilation"—two things that destroy each other.

27. Why is this a problem? If Yellow ★ Pink = Black and Yellow ★ Orange = Black, can you tell what Yellow was multiplied by if the answer is Black?

Part 5: Three Types of ★ Equations

When solving ★ equations, three different things can happen:

- Exactly ONE answer (like ⊕ equations)
- NO answer at all (impossible!)
- MORE THAN ONE answer (multiple solutions!)

Type 1: Exactly One Answer

28. Green \star ? = Pink \rightarrow Answer: _____ (Check: does it work?)

29. White \star ? = Orange \rightarrow Answer: _____

Type 2: No Answer At All

30. Yellow \star ? = White

Search the entire Yellow row. Is White anywhere in that row? YES / NO

Conclusion: This equation has _____ solution!

Type 3: More Than One Answer

31. Yellow \star ? = Black

Find one answer: _____

Keep looking in the Yellow row... is there ANOTHER answer? _____

This equation has MULTIPLE solutions!

Practice: Sort These Equations

For each equation, determine: ONE answer, NO answer, or MULTIPLE answers

32. Pink \star ? = White _____

33. Green \star ? = Yellow _____

34. Orange \star ? = Black _____

35. White \star ? = Green _____



Discovery Question

36. Compare \oplus and \star : Which operation is more "reliable" for solving equations? Why?

★ Bonus Challenge

The "troublemaker" colors for ★ were: Yellow (2), Pink (3), and Orange (4).

The "well-behaved" colors were: White (1) and Green (5).

Think about these number values and the number 6 (how many colors we have)...

37. What do 2, 3, and 4 have in common with 6?

Hint: Think about what numbers divide evenly into these values.

38. What do 1 and 5 have in common?

39. Prediction: If we had a different number of colors, would some tables work perfectly (every color has an inverse, no annihilation)?

□ Looking Ahead

In the next worksheet, we'll explore tables with different numbers of colors.

You'll discover which numbers make "perfect" tables—and this pattern

turns out to be one of the most important ideas in all of mathematics!