When Your Old Math Tricks Stop Working

Upgrading your mental models

Name: _____ Date: _____

Part 1: Your Old Models Were Great! (For certain numbers...)

"Take away" and "split into groups" worked perfectly — when you only had positive whole numbers.

But now try these. Circle POSSIBLE or IMPOSSIBLE for the old model:

1. "Take away" model: You have 8 cookies. Take away 3.
   POSSIBLE / IMPOSSIBLE
2. "Take away" model: You have 4 apples. Take away 7.
   POSSIBLE / IMPOSSIBLE
3. "Split into groups" model: Split 12 candies into 4 groups.
   POSSIBLE / IMPOSSIBLE
4. "Split into groups" model: Split 8 cups into ½ of a group.
   POSSIBLE / IMPOSSIBLE

💡 **But wait! In the REAL WORLD...**

5. You have $4. You borrow $7. What's your balance? _____
6. How many ½-cup scoops can you get from 8 cups of ice cream? _____

The situations are REAL — we just need better mental models!

Part 2: When "Bigger" and "Smaller" Intuition Fails

You used to believe: "Addition makes bigger. Multiplication makes MUCH bigger."
That was true for positive whole numbers. But now...

Calculate each. Then circle whether the answer is BIGGER or SMALLER than the first number:

7. $10 + (-4) =$ _____ BIGGER / SMALLER than 10
8. $6 + (-9) =$ _____ BIGGER / SMALLER than 6

9. $12 \times \frac{1}{2} =$ _____ BIGGER / SMALLER than 12

10. $8 \times (\frac{1}{4}) =$ _____ BIGGER / SMALLER than 8

11. $6 \div \frac{1}{2} =$ _____ BIGGER / SMALLER than 6

12. $10 \div (\frac{1}{5}) =$ _____ BIGGER / SMALLER than 10

🔍 Discovery Question

13. Can you still use "bigger or smaller" to guess which operation to use? YES / NO
What should you do instead? _____

Part 3: Tricky Word Problems — Trust Structure, Not Intuition!

For each problem: (a) Guess if the answer will be bigger or smaller. (b) Solve it. (c) Check!

14. A photo is 18 inches wide. You scale it to ⅓ the size. How wide is it now?

(a) Will the answer be bigger or smaller than 18? BIGGER / SMALLER

(b) What operation? + / − / × / ÷ Calculation: _____

(c) Answer: _____ inches Was your prediction correct? YES / NO

Hint: Scaling IS multiplication, even when things get smaller!

15. How many ¼-pound burgers can you make from 6 pounds of beef?

(a) Will the answer be bigger or smaller than 6? BIGGER / SMALLER

(b) What operation? + / − / × / ÷ Calculation: _____

(c) Answer: _____ burgers Was your prediction correct? YES / NO

Hint: "How many quarters fit in 6?" — division can make things bigger!

16. You have $5 in savings. You add a debt of $8. What's your balance?

(a) Will the answer be bigger or smaller than 5? BIGGER / SMALLER

(b) What operation? + / − / × / ÷ Calculation: _____

(c) Answer: $ _____ Was your prediction correct? YES / NO

Hint: Adding a debt means adding a negative number!

17. A recipe uses 2 cups of flour. You want to make 1½ times the recipe.

(a) Will the answer be bigger or smaller than 2? BIGGER / SMALLER

(b) What operation? + / − / × / ÷ Calculation: _____

(c) Answer: _____ cups Was your prediction correct? YES / NO

Hint: "Times" means multiply — and multiplying by more than 1 DOES make bigger!

🔍 Key Insight

18. Complete this rule: "Multiplying by a number greater than 1 makes things _____.
Multiplying by a number between 0 and 1 makes things _____."

Part 4: The PEMDAS Problem — And How Inverses Fix It

Many students think PEMDAS means: M before D, and A before S. **That's WRONG!**
M and D are the same level (left to right). A and S are the same level (left to right).

First, solve these the WRONG way (doing A before S, or M before D):

19. WRONG: 10 − 4 + 3 = 10 − 7 = _____ (This is incorrect!)

20. WRONG: 12 ÷ 3 × 2 = 12 ÷ 6 = _____ (This is also incorrect!)

Now solve them the RIGHT way (left to right within each level):

21. RIGHT: 10 − 4 + 3 = _____ + 3 = _____

22. RIGHT: 12 ÷ 3 × 2 = _____ × 2 = _____

✨ The Magic Fix: Rewrite Using Inverses!

When there's no subtraction or division, order within a level doesn't matter!

23. Rewrite: 10 − 4 + 3 = 10 + (_____) + 3 = _____

24. Rewrite: 12 ÷ 3 × 2 = 12 × (_____) × 2 = _____

Hint: Replace "− 4" with "+ (−4)" and "÷ 3" with "× (⅓)"

Rewrite each expression using only + and ×, then solve:

25. 8 − 5 + 2 − 1 = 8 + (_____) + 2 + (_____) = _____

26. 20 ÷ 4 × 2 ÷ 5 = 20 × (_____) × 2 × (_____) = _____

Part 5: The Big Picture

📝 Reflection

27. Your old mental models ("take away," "split into groups") were like training wheels. Why were they helpful when you were younger?

_____

28. Why do you need a new mental model now?

_____

⭐ Remember

You're not starting over — you're leveling up!

Using inverses with operations that follow the commutative, associative, and distributive properties gets your brain structured for algebra. That's why it feels easier!