

The Mayan Numeral System

Journey to Ancient Mesoamerica — 300–900 AD

★ ANSWER KEY — TEACHER COPY ★

Part 1: Time Travel Challenge

Fill-in-the-blank — accept reasonable variations

1. The Maya lived in **Mesoamerica / Mexico and Guatemala**
2. Folded books called **codices (codex)**, made from bark
3. Numbers were written **vertically**
4. First to invent a symbol for **zero**

Part 2: Meet the Three Symbols

Students describe the symbols — accept drawings or written descriptions

5. 4 → **4 dots (no bars)**
6. 8 → **1 bar on bottom + 3 dots on top**
7. 13 → **2 bars on bottom + 3 dots on top**
8. 19 → **3 bars on bottom + 4 dots on top**

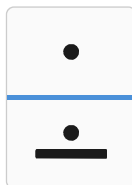
Teacher note — Part 2: Verify that students understand dots sit ON TOP of bars, never below. The number 8 is the key diagnostic: a student who writes three bars and one dot likely has the orientation backwards. The full 0–19 reference grid in the student worksheet shows the correct layout for every number.

Part 3: The Place-Value System

Part 3 is a reading / concept section with no fill-in blanks. The worked examples below are for teacher reference.

Worked examples — for teacher reference:

Example 1 — 26



20s place (top): 1 dot = $1 \times 20 = 20$
1s place (bottom): 1 bar + 1 dot = $5 + 1 = 6$

$$20 + 6 = 26 \checkmark$$

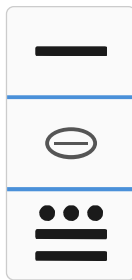
Example 2 — 118



20s place (top): 1 bar = $5 \times 20 = 100$
1s place (bottom): 3 bars + 3 dots = $15 + 3 = 18$

$$100 + 18 = 118 \checkmark$$

Example 3 — 2,013



400s place (top): 1 bar = $5 \times 400 = 2,000$

20s place (middle): shell = $0 \times 20 = 0$

1s place (bottom): 2 bars + 3 dots = $10 + 3 = 13$

$2,000 + 0 + 13 = 2,013 \checkmark$

Part 4: Mayan → Our Numbers

Reading Mayan → base-10:

- 20s: 2 dots | 1s: 1 bar + 3 dots → $(2 \times 20) + (8) = 40 + 8 = 48$ [1 bar + 3 dots = $5+3 = 8$]
- 20s: 1 bar + 1 dot | 1s: shell → $(6 \times 20) + 0 = 120$ [1 bar + 1 dot = $5+1 = 6$ twenties]
- 400s: 1 dot | 20s: shell | 1s: 2 dots → $(1 \times 400) + 0 + 2 = 402$
- 400s: 1 dot | 20s: 3 dots | 1s: 2 bars + 1 dot → $(1 \times 400) + (3 \times 20) + 11 = 471$ [2 bars + 1 dot = $10+1 = 11$]

Converting our numbers to Mayan notation:

- $45 \rightarrow$ 20s: **2 dots** | 1s: **1 bar** ($45 \div 20 = 2 \text{ r } 5$; 5 = one bar)
- $380 \rightarrow$ 400s: **shell (0)** | 20s: **3 bars + 4 dots (= 19)** | 1s: **shell (0)** ($380 \div 400 = 0$; $380 \div 20 = 19 \text{ r } 0$)
- $2,045 \rightarrow$ 400s: **1 bar (= 5)** | 20s: **2 dots** | 1s: **1 bar** ($2045 \div 400 = 5 \text{ r } 45$; $45 \div 20 = 2 \text{ r } 5$; 5 = one bar)

Teacher note — Q14: 380 is a great diagnostic. Because $380 < 400$, the 400s place gets a shell. Students must still write the shell — omitting it would make the number unreadable. The 20s place holds 19 (= 3 bars + 4 dots, the maximum), and the 1s place also gets a shell.

Part 5: The Zero Hero

- Without the shell in the 20s place, the reader would see only two places and likely interpret it as **213** (= 1 bar in the 20s place $\times 20$, plus 2 bars + 3 dots in the 1s). *Accept any answer that identifies the ambiguity caused by the missing zero.*
- Babylonians used **spacing** (blank space). Maya used a **shell symbol**.
- Open-ended — look for:** In a vertical system, a missing place could cause all the places above it to collapse down, making an entirely different number. A visible shell guarantees the reader knows a place exists but is empty.

Part 6: Why Base-20?

19. *Open-ended* — look for: 10 fingers + 10 toes = 20 body parts to count on, making 20 a natural grouping size. Accept any answer connecting body counting to base selection.

20. Max two-place value: **399** ($19 \times 20 + 19 = 380 + 19 = 399$)

Part 7: The Living Legacy

21. Solar calendar: **365** days
22. Sacred calendar: **260** days
23. Observations made by the **Maya** over **1,000–2,000** years ago. (Accept any reasonable number of years consistent with 300–900 AD.)

Part 8: System Comparison

Completed comparison table:

Feature	Our Base-10 System	Mayan Base-20 (answers)
Base number	10	20
Core symbols	0–9 (ten symbols)	Dot, Bar, Shell (3 symbols)
Has zero?	Yes	Yes (shell symbol)
Place-value system?	Yes	Yes
Direction places grow	Left to right	Bottom to top
Max value in one place	9 (then carry)	19 (then carry)
Main strength	Easy (10 fingers)	Fingers & toes (20); only 3 symbols needed

Part 9: Challenge Problems

Worked solutions:

Q24 — $14 + 9 = ?$

$14 + 9 = \mathbf{23}$

$23 \div 20 = 1$ remainder 3

Mayan: 20s place = **1 dot** | 1s place = **3 dots**

(No bars needed in either place for this answer)

Q25 — $17 + 16 = ?$

$17 + 16 = \mathbf{33}$

$33 \div 20 = 1$ remainder 13

Mayan: 20s place = **1 dot** | 1s place = **2 bars + 3 dots** (= $10+3 = 13$)

Q26 — top = 1 bar | middle = shell | bottom = 2 bars + 3 dots

400s place (top): 1 bar = $5 \times 400 = 2,000$

20s place (middle): shell = $0 \times 20 = 0$

1s place (bottom): 2 bars + 3 dots = $10 + 3 = 13$

Total: $2,000 + 0 + 13 = 2,013$

(This is the same as Example 3 on the worksheet — intentional! Students who decoded Example 3 correctly should recognize it.)

Part 10: Reflection

Open-ended — grade on quality of reasoning, not specific content

Q27 — What surprised you most?

Strong answers may mention: only 3 symbols needed; numbers grow upward instead of sideways; zero existed 1,500+ years ago in the Americas; how large numbers can be represented with simple dots and bars.

Q28 — Bottom-to-top vs. left-to-right:

Similar: both are place-value systems; both carry when a place is full; both use zero as a placeholder.

Different: direction of reading; Maya uses only 3 symbols vs. our 10; Maya places carry at 20, ours at 10; within each Maya place, dots stack above bars (a second layer of additive notation).

Q29 — Question for a Mayan astronomer:

Any thoughtful question accepted. Look for curiosity about: how they tracked planets without telescopes, why they chose base-20, how they discovered the 365-day year, what zero meant to them conceptually.